OPTIMAL NUMERICAL MODEL OF A NON-STATIONARY HEAT TRANSFER THROUGH A WALL

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Summary:

The problems of steady-state and transient heat conduction for a given geometry can be solved analytically and numerically. While the use of analytical solutions is limited, numerical methods can be used to solve heat transfer problems in complex geometries with more intricate boundary conditions, using computer simulations. Complex geometries are discretized to form an efficient numerical mesh for solving the given problem. This paper focuses on the calculation of one-dimensional, transient heat transfer for a wall with a thickness of 4 cm. The wall temperatures are calculated for each mesh node at a given moment in time. Two types of analyses were performed, using FSM analysis (Finite Strip Method) and FEM analysis (Finite Element Method). The former was conducted using Microsoft Excel, while the latter was calculatedusing ANSYS software. A parametric study was performed in order to analyse the influence of spatial and temporal step size on the accuracy of the solution. Finally, the optimal solution was determined to obtain temperature results with the lowest relative error within the wall nodes, while maintaining the efficiency of the computational model.

Key words: heat conduction, FSM analysis, FEM analysis, wall, ANSYS

NUMERIČKI MODEL NESTACIONARNOG PROVOĐENJA TOPLOTE KROZ ZID

Rezime:

Problemi stacionarnog i nestacionarnog provođenja toplote za zadatu geometriju mogu se rešiti numerički i analitički. Kako je upotreba analitičkog rešenja ograničena, numeričke metode se mogu koristiti za rešavanje problema prenosa toplote kod složenih geometrija sa specifičnim graničnim uslovima, primenom računara. Složene geometrije se diskretizuju radi formiranja efikasne numeričke mreže za rešavanje datog problema. Rad se bazira na proračunu za jednodimenzionalno, nestacionarno provođenje toplote za zid debljine 4 cm. Temperatura zida je sračunata u svakom čvoru mreže u datom vremenskom trenutku. Korišćene su dve analize: FSM analiza (Metod konačnih traka) i FEM analiza (Metod konačnih elemenata). Prva analiza je sprovedena korišćenjem programskog paketa Microsoft Excel, dok je druga sprovedena korišćenjem ANSYS softvera. Izvršena je parametarska studija kako bi se analizirao uticaj prostornog i vremenskog koraka na tačnost rešenja. Na kraju, optimalno rešenje je određeno kako bi se dobili rezultati temperature sa najmanjom relativnom greškom unutar čvorova zida, uz očuvanje efikasnosti računskog modela.

Ključnereči: provođenjetoplote, FSM analiza, FEM analiza, zid, ANSYS

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1. INTRODUCTION

The laws of heat transfer are crucial in designing and operating various forms of steam generators, furnaces, preheaters, heat exchangers, coolers, evaporators, and condensers in numerous industries [1]. For the heat transferto occur, there must be a temperature difference. This difference can exist between parts of a single body or between two or more uninsulated bodies. Heat transfer consists of a series of distinct processes divided into three groups: heat conduction (conduction), heat transfer through a fluid (convection), and heat radiation (radiation). Figure 1 illustrates the processes of heat transfer.In addition to this classification, heat transfer can be categorized as steady-state and transient. In steady-state heat transfer, the temperatures of individual points within a body do not change with time (although they may vary spatially). In contrast, in transient heat transfer, these temperatures change with time [2]. If the temperature changes over time, energy is stored within the body or transferred to another body.
Heat conduction problems, both stationary and transient, for a given geometry, can be solved analytically and numerically. Analytical solutions apply to "simpler" geometric domains with straightforward boundary conditions.Numerical solutions for heat conduction problems are employed in cases of complex geometries with intricate boundary conditions, necessitating computer-based approaches when analytical solutions are not feasible. Complex geometries are approximated to create an efficient numerical grid for solving the given problem. A numerical mesh is formed within the observed geometric domain, where there are n unknown temperatures. To determine these n unknowns, systems of algebraic equations with n unknowns must be established. Solving these systems of algebraic equations yields the temperature distribution (temperature profile) within the observed geometric domain. The obtained temperature values are often referred to as temperature values at discrete points, while the mesh points are commonly called numerical nodes.In the scope of this work, numerical methods for solving heat conduction problems were primarily analyzed [3], while analytical solutions for heat conduction problems can be found in the literature [4].
The study is focused on the calculation of one-dimensional, transient heat conduction for a 4 cm thick wall. The wall mesh consists of nodes, and for each of these nodes, temperatures need to be computationally determined. Two analyses were employed: FSM analysis (Finite Strip Method) and FEM analysis (Finite Element Method) to obtain temperature values within the wall and at the wall's end. Regarding the spatial step, values of 0.02 m, 0.01 m, and 0.005 m were used, while for the time step, the chosen values were 12s, 6s, 3s, 1.5s, 1s, 0.5s, and 0.25s.By comparing the two calculation methods in a specific example, it was determined that the obtained solution accuracy is satisfactory.
The numerical methods used for solving heat conduction problems include:

- Finite Element Method (FEM),
- Finite Difference Method (FDM),
- Finite Strip Method (FSM),
- Boundary Element Method,
- Energy Balance Method for Control Volume.

2. FSM-FEM ANALYSIS

Numerical analysis for a 4 cm thick wall was conducted using the ANSYS software. To solve this problem numerically, the Finite Difference Method was used. First, it is necessary to set up a discrete grid and consider how temperature evolves, using the heat conduction differential equation. Setup:

- Wall thickness, L = 0.04 m,
- Thermal conductivity, k = 28 W/mK,
- Thermal diffusivity, a = 12.5 x 10^{-6} m^2/s,
- Initial wall temperature, $T_{initial}$ = 23 °C = 296 K,
- Heat source within the wall, e_{dot} = 5 x 10^6 W/m^3,
- Ambient temperature on one side of the wall, $T_{ambient}$ = 20 °C = 293 K,
- Convective component on the other side of the wall, $T_{infinity}$ = 30 °C = 303 K,
- Heat transfer coefficient, h = 45 W/m^2K.

The numerical grid has three nodes: the left node, the central node, and the right node. The central node will be in the middle of the wall thickness, and the left and right nodes will be at the boundaries of the wall.During the calculation, both the time and spatial steps were altered. With each change made in comparison to the initial example, different values for the Fourier number were obtained. The time duration of the numerical analysis was limited to one minute to facilitate a more straightforward comparison with the results obtained analytically, as the computational results were generated for oneminute.

2.1. FINITE STRIP METHOD (FSM)

By applying the Finite Strip Method, temperature values within the nodes of the numerical grid for a wall with a thickness of 4 cm were obtained. The analysis was conducted by varying both the time and spatial steps. The temperature monitoring period was set to conclude after one minute for analytical calculation. The following Figures (1-3) visually represent the temperatures obtained at 12-second intervals, with varying time and spatial step parameters.

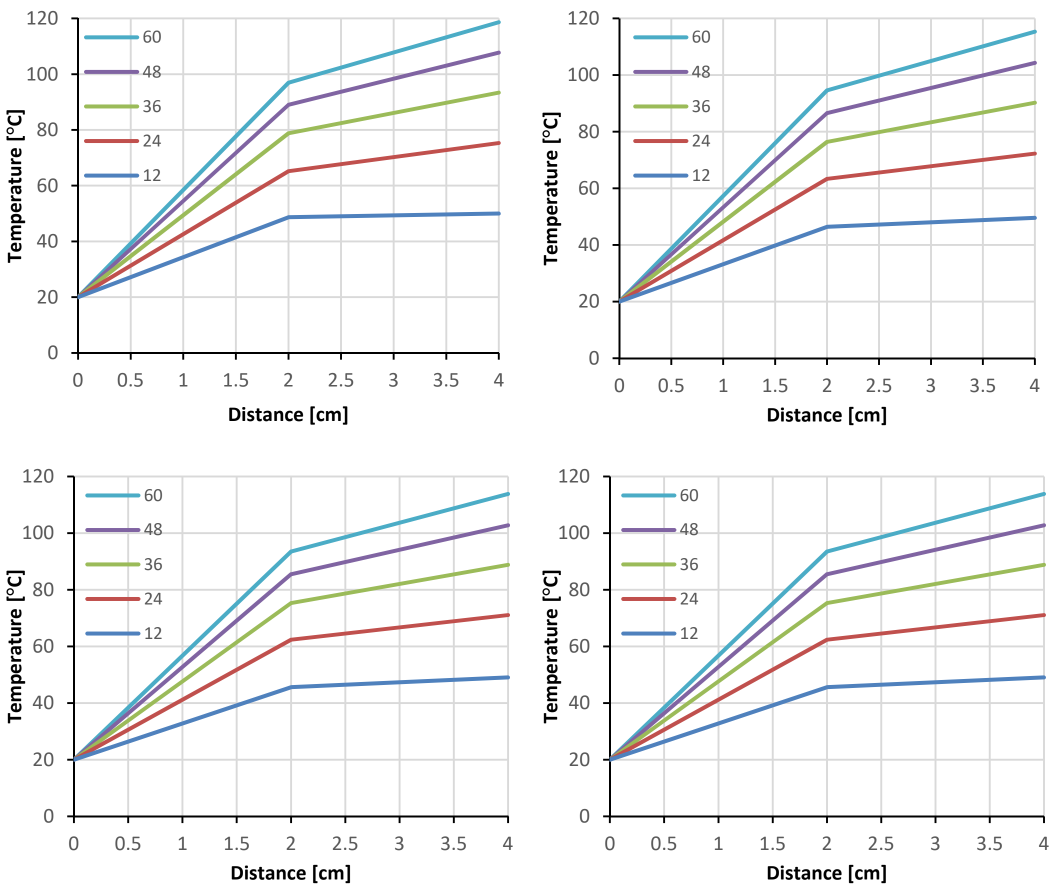

Fig. 1Temperatures within the nodes of a 4 cm thick wall, using a spatial step of 0.02 m and a time step of 12s (top left), 6s (top right), 3s (bottom left), and 1.5s (bottom right)

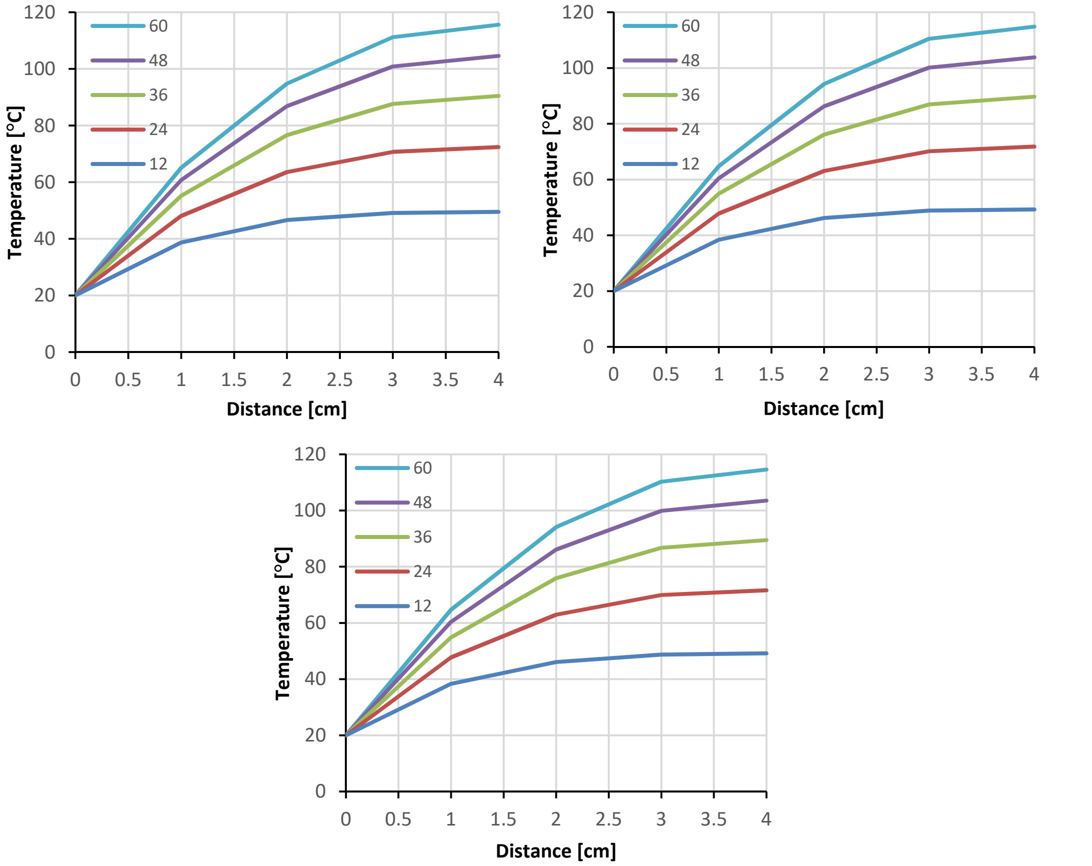

Fig. 2 Temperatures within the nodes of a 0.04 m thick wall, using a spatial step of 0.01 m and a time step of 3s (top left), 1.5s (top right), 1s (bottom)

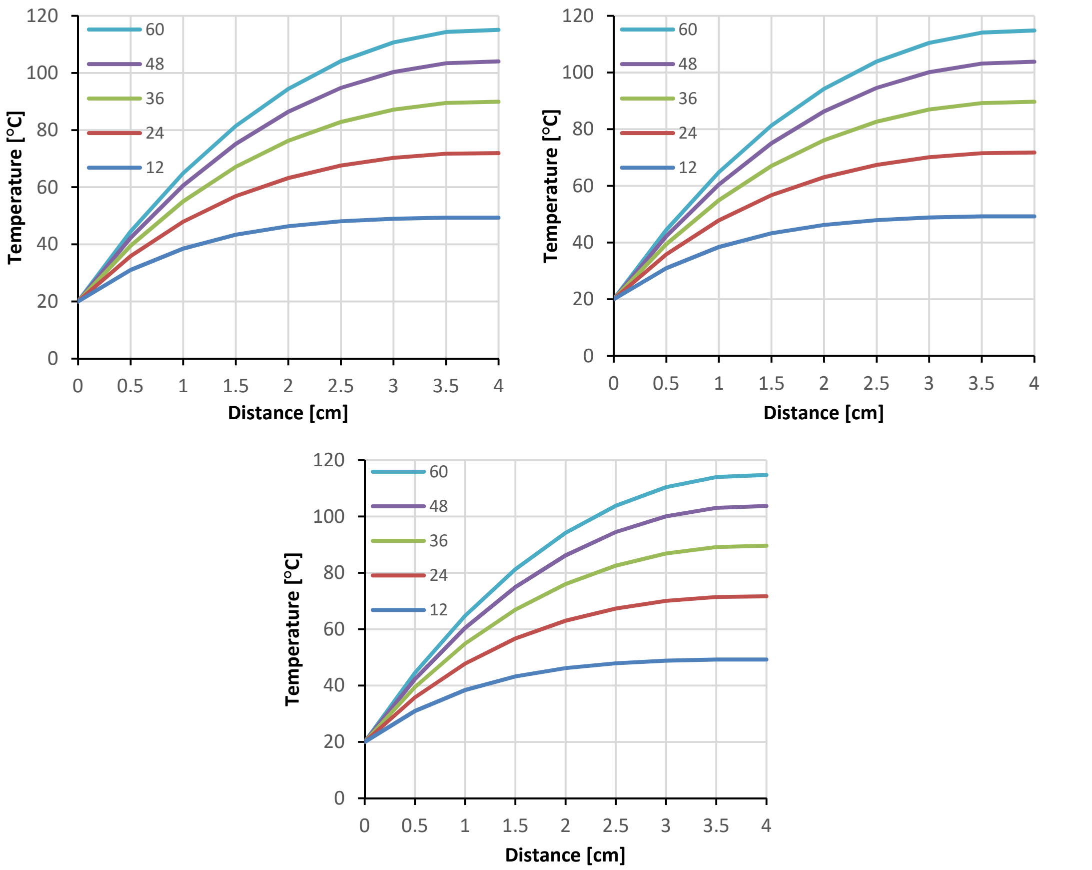

Fig. 3 Temperatures within the nodes of a 0.04 m thick wall, using a spatial step of 0.005 m and a time step of 1s (top left), 0.5s (top right), 0.25s (bottom)

Based on the previous diagrams, it can be concluded that using a denser numerical grid and setting lower values for the time step results in a smoother temperature curve.

2.2. FINITE ELEMENT METHOD (FEM)

In the next step, a parametric analysis was conducted using the Finite Element Method (FEM Analysis). The analysis was also performed for some time lasting one minute, and it focused on a 4 cm thick wall. Figure 4 illustrates curves based on the number of elements, with temperature values observed at the centre of the wall (central node) and the end of the wall (last node) over one minute.

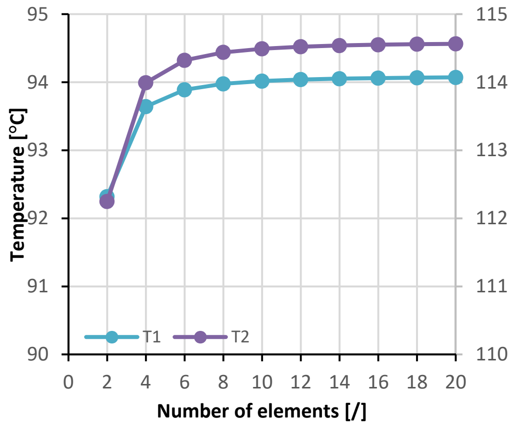

Fig. 4 Temperatures obtained at the central node and the node located at the end of the wall as a function of the number of elements

Figure 5 shows the relative error, expressed in percentages, for temperatures obtained at the central node and the node located at the end of the wall as a function of the number of elements (ranging from 2 to 20 elements).

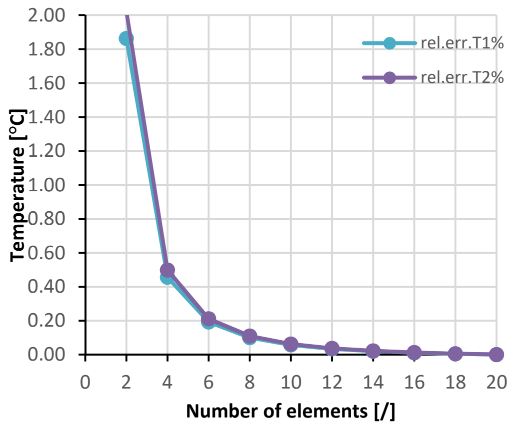

Fig. 5 Relative error for temperatures obtained at the central node and the node located at the end of the wall as a function of the number of elements

From Figure 5, it can be observed that the relative error is higher when the number of elements is lower (2 elements). Then, already at the number of elements 4, this error sharply decreases, and such a trend continues further until the number of elements 18, where the value of the relative error is 0%. Therefore, as the number of elements increases, more accurate temperature values at the mesh nodes are calculated.

In the following Figures (6-8), the relative error is shown, with variations in time and spatial step. The observation period is one minute, and relative errors are displayed every 12 seconds. The subject of analysis is a 4 cm thick wall. In Figure6 (left), relative errors are shown during numerical analysis for the temperature within the central node of a 0.04 m thick wall, using a spatial step of 0.02 m, with variations in the time step (12s, 6s, 3s, and 1.5s). The number of elements is 2. From the figure, it is evident that the smallest error was achieved using a time step of 6 seconds. Figure 6 (right) presents the same scenario as shown in the previous diagram, but it observes temperatures at the end of the wall (last node). When observing temperatures at the end of the wall, it can be seen that the smallest relative error is achieved using time steps of 3 seconds and 6 seconds.

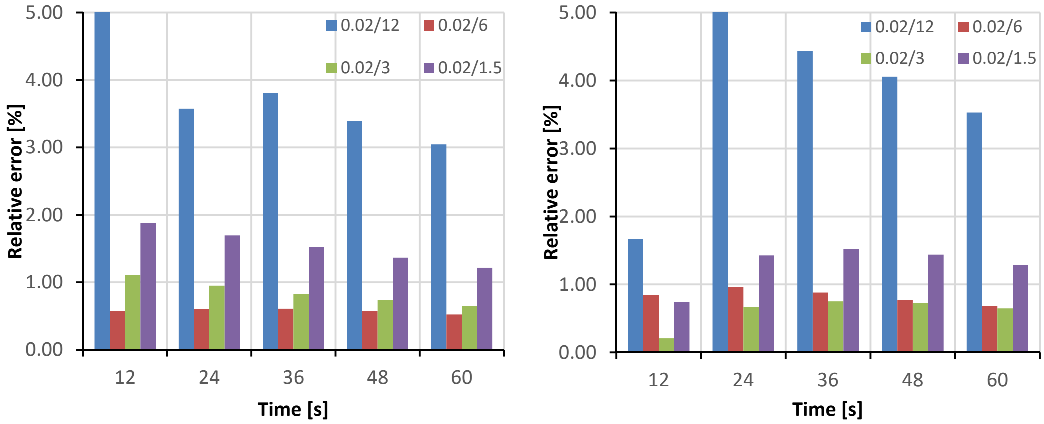

Fig. 6 Relative error for temperatures obtained at the centre of the wall (left) and at the end of the wall (right) using a spatial step of 0.02 m, with time steps of 12s, 6s, 3s, and 1.5s

In Figure 7 (left), relative errors during numerical analysis for the temperature within the central node of a 0.04 m thick wall are depicted. It uses a spatial step of 0.01 m, with variations in the time step (3s, 1.5s, and 1s). The number of elements is 4. Figure 7 (right) refers to temperatures obtained at the end of the wall. From Figure7, it is evident that the smallest error is achieved using a time step of 1 second. Although the smallest error is obtained with a time step of 1 second, it can be noticed that using time steps of 3s or 1.5s results in a relative error value below 1%, which can be considered satisfactory.

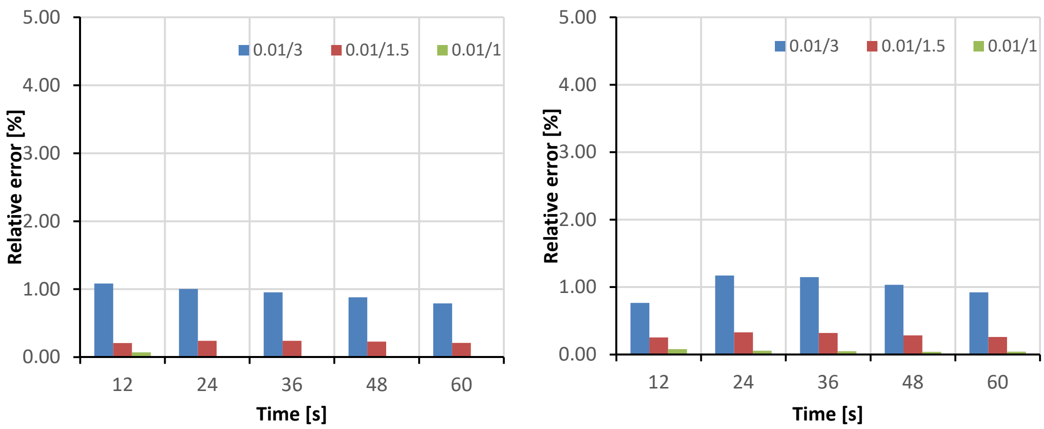

Fig. 7 Relative error for temperatures obtained at the centre of the wall (left) and at the end of the wall (right) using a spatial step of 0.01 m, with time steps of 3s, 1.5s, and 1s

In Figure 8 (left), relative errors during numerical analysis for the temperature within the central node of a 0.04 m thick wall are shown. It uses a spatial step of 0.005 m, with variations in the time step (1s, 0.5s, and 0.25s). The number of elements is 8. From

Figure8, it is clear that the smallest error is achieved using a time step of 0.25 seconds. Nevertheless, using time steps of 0.5s and 1s still provides fairly accurate values since their relative error values are below 1%.

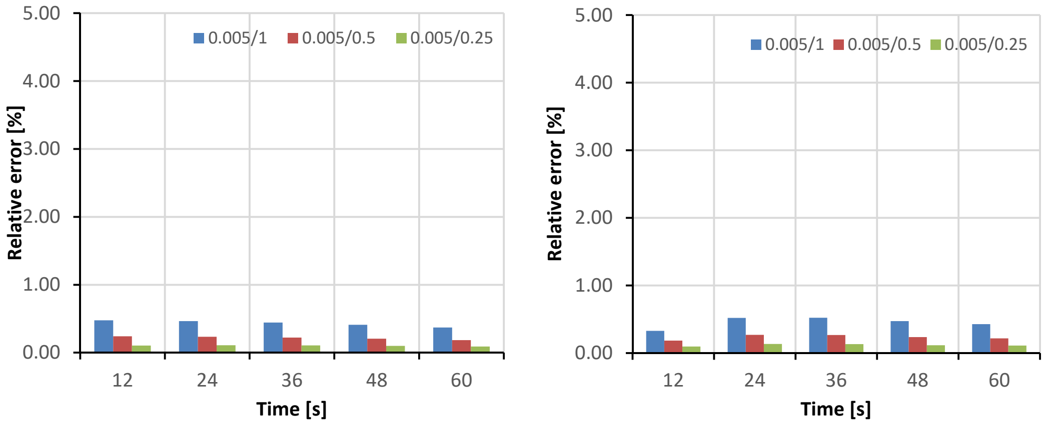

Fig. 8 Relative error for temperatures obtained at the centre of the wall (left) and at the end of the wall (right) using a spatial step of 0.005 m, with time steps of 1s, 0.5s, and 0.25s

2.3. RELATIVE ERRORS OBTAINED USING FSM AND FEM METHOD

The following diagrams show the relative errors obtained using the Finite Strip Method (FSM) and the Finite Element Method (FEM). Temperatures within the nodes of the numerical grid for a 4 cm thick wall were analyzed. The varied parameters were spatial and time steps. The analysis was conducted over a one-minute duration. Before the analysis, it was adopted that any relative error valuebelow 1%, between two consecutive time step variations, represents satisfactory accuracy in obtaining temperatures within the numerical grid nodes.Figure9 (left) illustrates the relative error using a spatial step of 0.01 m and consecutive time steps (12s-6s, 6s-3s, 3s-1.5s). The number of elements for this example is 2, and temperatures were observed at the center of the wall. From Figure 9, it can be concluded that using a time step variation of 3s-1.5s results in a relative error below 1%. In Figure9 (right), significant oscillations in the relative error values can be observed depending on the moment of temperature observation (12s, 24s, 36s, 48s, and 60s), indicating that a larger time step increases the likelihood of result inaccuracies.

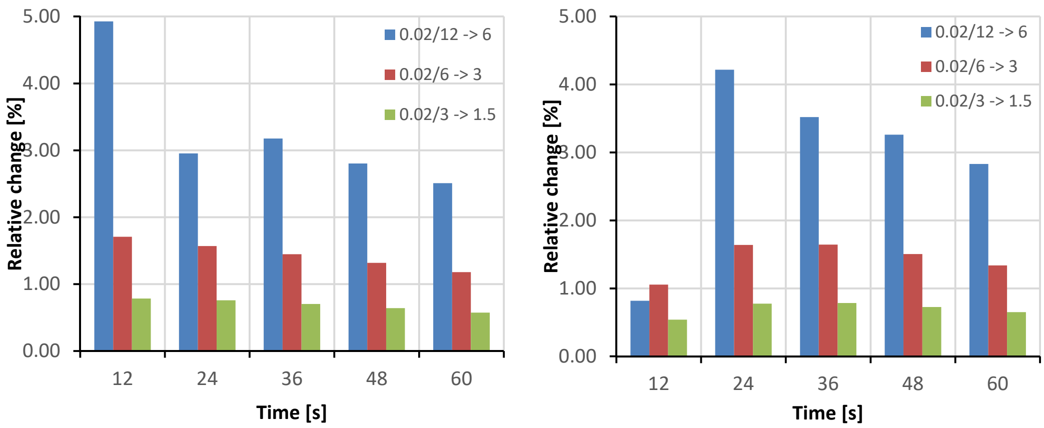

Fig. 9 Relative error using FSM and FEM analysis with a spatial step of 0.02 m, with time step variations (12s-6s, 6s-3s, 3s-1.5s) for temperatures observed at the center of the wall(left) and at the end of the wall (right)

In Figure10 (left), a spatial step of 0.01 m and a time step variations (3s-1.5s and 1.5s-1s), were used. It is visible that a smaller time step results in a lower relative error value. However, using time steps of (12s-6s and 6s-3s) still provides a relative error value below 1%, with 1% being considered as the acceptable threshold. In this example, 4 elements were used, and temperature values were observed at the center of the wall (Figure 10, left) and the end of the wall (Figure10, right).

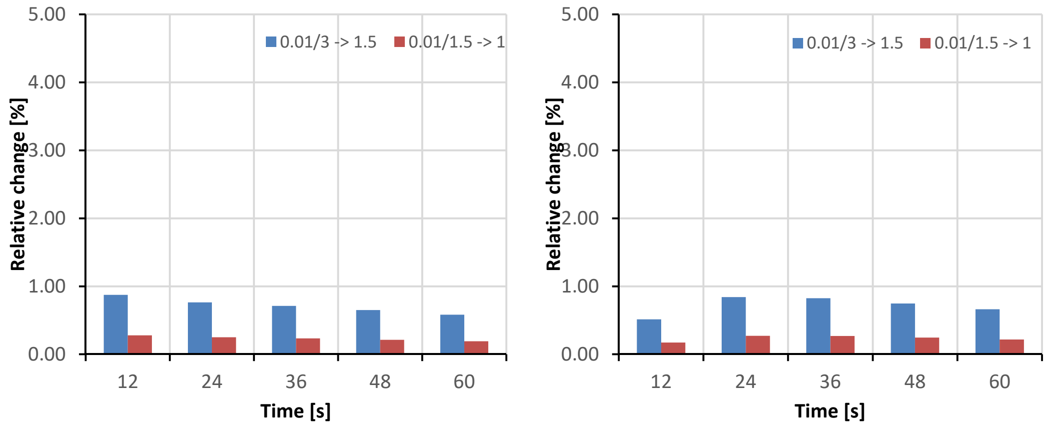

Fig. 10 Relative error using FSM and FEM analysis with a spatial step of 0.01 m, with time step variations (3s-1.5s and 1.5s-1s) for temperatures observed at the center of the wall(left) and at the end of the wall (right)

Figure11 (left) presents an example where a spatial step of 0.005 m was used, while the time step variation was (1s-1.5s and 0.5s-0.25s). A smaller time step resulted in a lower relative error value in both cases. However, in both scenarios, the relative error values were well below 1%, which is considered satisfactory. In this example, 8 elementswere used, and temperature values were observed at the center of the wall (Figure11, left) and the end of the wall (Figure11, right).

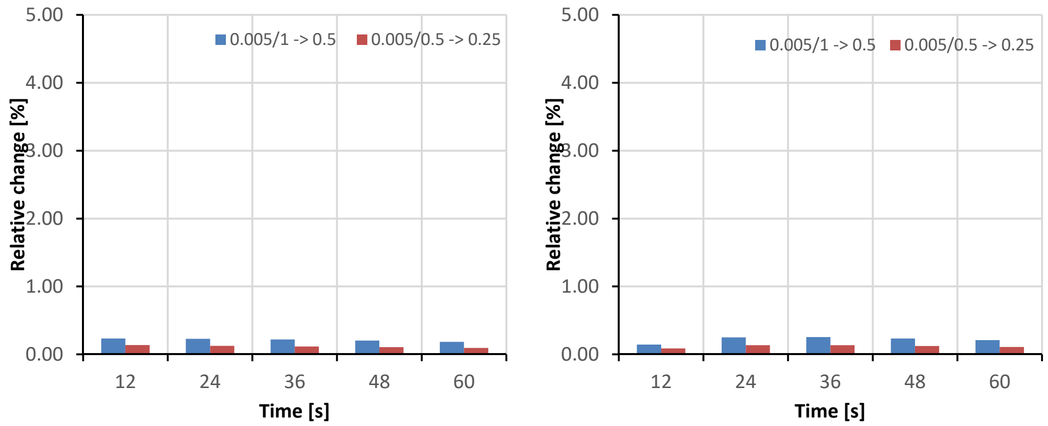

Fig. 11 Relative error using FSM and FEM analysis with a spatial step of 0.005 m, with time step variations (1s-0.5s and 0.5s-0.25s) for temperatures observed at the center of the wall(left) and at the end of the wall (right)

Figure12shows a comparative analysis of temperatures through a 4 cm thick wall using FSM and FEM analysis. The analysis was conducted at time intervals of 12s, 24s, 36s, 48s, and 60s.

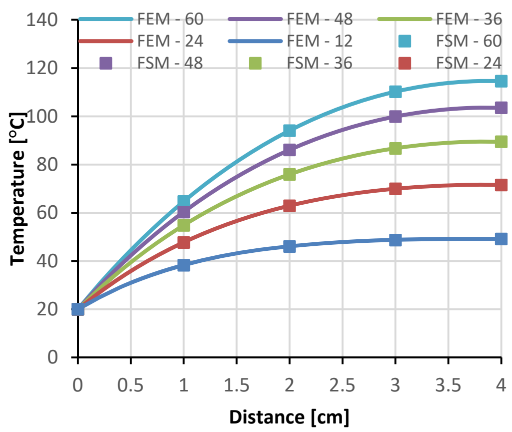

Fig. 12 Comparison of temperature results obtained for a 4 cm thick wall using FSM and FEM methods at 12s, 24s, 36s, 48s, and 60 seconds

3. CONCLUSION

The paper presents an example of one-dimensional, transient heat conduction through a 4 cm thick wall divided into three nodes. Temperature values for each of these points were calculated using the Finite Strip Method (FSM), with a time step of 0.2 minutes. The calculation was stopped after obtaining temperature values after one minute. Solution convergence was monitored through sensitivity analysis of the time and spatial step parameters until optimal values were determined in terms of solution accuracy and computation time. Additionally, as an external verification of the computational model, a model was created in the ANSYS software based on the Finite Element Method (FEM). By comparing the results of the two calculation methods in this specific example, it was determined that the achieved accuracy of the solution was satisfactory.
Regarding the spatial step, three values were used: 0.02 m, 0.01 m, and 0.005 m, and for the time step, the following values were considered: 12s, 6s, 3s, 1.5s, 1s, 0.5s, and 0.25s. After conducting all the analyses and variations, it was concluded that the best approach is to use a spatial step of 0.01 m and a time step of 1s to obtain values with the smallest relative error for temperatures within the nodes of the wall. The obtained values for the spatial and time steps are considered optimal, and further refinement of the numerical grid would increase the number of spatial points, potentially leading to rounding error accumulation, which can result in an increased error accumulation.

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