TIPOLOGIJA NSPA *PUSHOVER* KRIVIH I POVRŠI ZA 3D SEIZMIČKI ODGOVOR KONSTRUKCIJA PREMA PERFORMANSAMA

TYPOLOGY OF NSPA *PUSHOVER* CURVES AND SURFACES FOR 3D PERFORMANCE-BASED SEISMIC RESPONSE OF STRUCTURES

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1 UVOD

Metodologija analize konstrukcija za uslove dejstva zemljotresa, kada su u pitanju praktični inženjerski problem, bazira se na primeni spektralno-modalne analize ili ekvivalentne statičke metode, dok se znatno manje primenjuje dinamička analiza [15]. Pored postojećih metoda, razvijene su i unapređuju se nove metode koje se baziraju na nelinearnom ponašanju konstrukcija u uslovima dejstva zemljotresa [16]. Integracija novih metoda sprovedena je u metodologiji analize konstrukcija prema seizmičkim performansama (PBEE − *Performance-Based Earthquake Engineering*), a koja se sprovodi za jednu komponentu ili veći broj komponenti seizmičkog dejstva, kao što su dve horizontalne i jedna vertikalna. Kada se primenjuje jedna komponenta seizmičkog dejstva, tada su numerički modeli konstrukcija 2D ravanski, 3D prostorni ili dekomponovani 3D modeli, a ukoliko se primenjuju dve ili tri komponente seizmičkog dejstva, tada su numerički modeli konstrukcija 3D prostorni modeli. Seizmički odgovor konstrukcija prema performansama u nelinearnom domenu moguće je prezentovati primenom *pushover* krivih za slučaj razmatranja jedne ili dve komponente seizmičkog dejstva. Primena *pushover* krivih kod 2D modela konstrukcija predstavlja standard za prezentaciju odgovora sistema u kapacitativnom domenu. U svim dosadašnjim istraživanjima nelinearnog ponašanja 3D modela konstrukcija u uslovima dejstva zemljotresa, prikazivanje odgovora sistema, u kapacitativnom domenu, sprovedeno

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1 INTRODUCTION

Methodology of the earthquake structural analysis, used in the everyday and routine engineering work, is based upon the multimodal spectral analysis or the equivalent static analysis, while the full dynamic earthquake analysis is applied in a smaller amount [15]. Besides these existing methods, various new ones are being developed and improved, based upon the nonlinear behaviour of structures when exposed to earthquake actions [16]. Integration of new methods has been achieved in the methodology of structural analysis according to seismic performance (PBEE - Performance-Based Earthquake Engineering), which is conducted for one or more components of the seismic action, such as the two horizontal and one vertical. When one component of the seismic action is applied, then the numerical models of structures are 2D planar, 3D spatial or decomposed 3D models, and if two or three components of the seismic action are applied, then the numerical models of structures are 3D spatial models. Performance-based seismic response of the structure in the nonlinear domain may be presented by a pushover curve when taking into consideration one or two components of the seismic action. The application of the pushover curves in 2D models of structures represents a standard for the presentation of the system response in the capacity domain. In all previous studies of the non-linear behaviour of the 3D structural models during the earthquake, the presentation of system response, within

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je analogno, kao i kod 2D modela − preko *pushover* krivih. Dodatno su kod 3D modela konstrukcija razmatrani i inkorporirani u analizu efekti bidirekcionog dejstva zemljotresa, torzije, neregularnosti u osnovi i po visini konstrukcije. U radovima [14] i [19] sprovedene su numeričke analize odgovora 3D modela zgrada, s tim što je odgovor sistema u kapacitativnom domenu razmatran za dva ortogonalna pravca i po dva smera za svaki pravac, kao što je prikazano i u [11], [12] i [13]. U radovima [21], [22] i [23] prikazana je modalna *pushover* analiza bazirana na proračunu skaliranjem dvokomponentalnih akcelerograma 3D modela zgrada, a kao odgovor prikazane su *pushover* krive za dva ortogonalna pravca. *Pushover* kriva dobija se iz NSPA analize koja se bazira na inkrementalno-iterativnoj proceduri, pri čemu se prati odgovor sistema sve do kolapsa [26], [27].

Seizmički odgovor konstrukcija prema performansama u nelinearnom domenu moguće je prikazati i primenom *pushover* površi za slučaj razmatranja dve komponente seizmičkog dejstva. Konstrukcija i generisanje *pushover* površi sprovodi se integracijom nelinearnih odgovora datih preko individualnih *pushover* krivih za različite uglove i za multikomponentalno dejstvo zemljotresa [4]. Prethodno je potrebno generisati multikomponentalne akcelerograme i spektre odgovora za različite uglove u odnosu na komponentu upravno na pravac pružanja raseda (*fault normal*) i komponentu paralelno pravcu pružanja raseda (*fault parallel*) [24], [25]. Pošto je primena *pushover* površi u prezentaciji i razmatranju perfomansi konstrukcija još u inicijalnoj fazi, otvorena su brojna pitanja, kao što su: detaljnija matematička formulacija, tipologija, aspekti primene na realnim modelima konstrukcija, specijalni slučajevi *pushover* površi, prednosti i nedostaci u prikazivanju performansi primenom *pushover* površi, itd. Predmet istraživanja u ovom radu jeste matematička formulacija i tipologija *pushover* krivih i površi za opšte modele konstrukcija zgrada. Sistematizacija tipova *pushover* krivih i površi izvršena je na osnovu obimnih iskustava autora na većem broju seizmičkih analiza numeričkih modela konstrukcija. Primenom *pushover* površi kod NSPA analize moguće je razmatrati i analizirati uticaje glavnih pravaca kod nesimetričnih zgrada, dok se kod dinamičke analize vremenskog odgovora, u zavisnosti od dominantnog pravca delovanja zemljotresa, razmatranje obično sprovodi preko akcelerograma [28]. Razmatranje nelinearnog odgovora sistema za multikomponentalno dejstvo zemljotresa moguće je još u fazi konceptualnog projektovanja vrednovanja projektnih rešenja [5]. Dinamičke analize mogućeg sudara susednih nesimetričnih višespratnih zgrada usled zemljotresa [29], kao i analize odgovora sistema s polukrutim ekscentričnim vezama u slučaju zemljotresa [7], ili analize efekta dejstva višestrukih zemljotresa [1] i dr., imajući u vidu da takve analize još nisu ugrađene u komercijalni softver, za sada nisu deo rutinskih inženjerskih aktivnosti u analizi uticaja zemljotresa.

the capacity domain, was done using the analogy with the 2D model − using a pushover curve. Additionally, bidirectional earthquake actions of the 3D models of a structure have been considered and incorporated in the analysis, as well as the torsion, irregularities in plan and along the height of the structure. In papers [14] and [19] numerical analyses of the responses of 3D building models are given, where the response of the system in the capacity domain has been considered for two orthogonal directions and two senses of each direction, as also shown in [11], [12] and [13]. In the papers [21], [22] and [23] the modal pushover analysis is given, based on calculation where scaling of two-component accelerograms of 3D building models was done, while the response is presented as the pushover curves for the two orthogonal directions. Pushover curve is obtained from the NSPA analysis which is based upon the incremental-iterative procedure, where the system response is followed all the way until the collapse [26], [27].

Seismic response of structures according to their performance in the nonlinear domain can be presented using the pushover surface by considering the two components of the seismic action. Construction and generation of the pushover surface is implemented by integrating the nonlinear responses given by individual pushover curves for different angles and multicomponent earthquake actions [4]. First, it is necessary to generate multicomponent accelerograms and response spectra for different angles relative to the component perpendicular to direction of the fault (fault normal) and a component in parallel to direction of the fault (fault parallel) [24], [25]. Since the application of the pushover surface in the presentation and consideration of structural performance is still at the initial stage, there are a number of questions which should be answered, such as: a more detailed mathematical formulation, typology, aspects of the application on real models of structures, special cases of pushover surfaces, advantages and disadvantages of using the pushover surface for presentation of seismic response, etc. The subject of research in this paper is the mathematical formulation and typology of pushover curves and surfaces for the general building structural models. Systematization of the types of pushover curves and surfaces is made according to the authors' valuable experience in a number of seismic analyses of numerical structural models. By considering the pushover surface using the NSPA analysis it is possible to consider and analyze the influence of the main directions of the non-symmetric buildings, while in dynamic analysis applying the time response and considering effects of the dominant earthquake direction, the analysis is normally conducted utilizing the accelerograms [28]. Consideration of the non-linear system response due to multicomponent earthquake action is possible even in the preliminary analysis [5]. Dynamic analysis of pounding of adjacent non-symmetric multi-story buildings during earthquakes [29], analysis of the seismic response of the system with semi rigid and eccentric connections [7], or analysis of the effects of multiple seismic actions [1] etc., are not a part of the routine engineering activities in earthquake analysis, since it is still unlikely that they are incorporated into the commercial software.

2 TIPOLOGIJA NSPA *PUSHOVER* KRIVIH

Analiza odgovora konstrukcija za dejstvo zemljotresa, pored vremenskog i frekventnog domena, razmatra se i u kapacitativnom domenu (CD), odnosno domenu analize kapaciteta nosivosti i deformacija konstrukcije. Odgovor 2D modela zgrada u kapacitativnom domenu prema nelinearnoj statičkoj *pushover* analizi, u opštem slučaju, definisan je granicama Γ_i (slika 1):

2 TYPOLOGY OF NSPA PUSHOVER CURVES

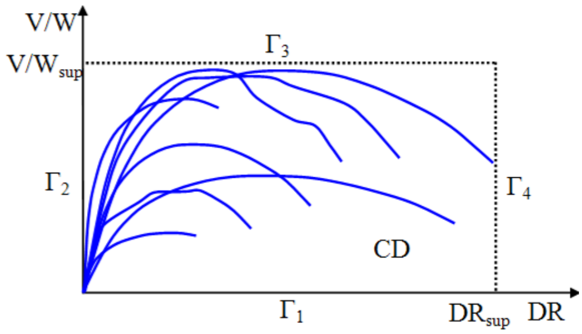
Analysis of the structural response to earthquake action, in addition to analysis in the time and frequency domains, is considered in the capacity domain (CD), i.e. the domain of the strength and deformation capacities of structures. Response of 2D building models in the capacity domain according to the nonlinear static pushover analysis is, in general, defined by boundaries Γ_i (Figure 1):

$$\mathrm{CD}:\begin{cases}\Gamma_1: DR\in[0,+\infty) & \wedge \quad V/W=0\\ \Gamma_2: \quad DR=0 & \wedge \quad V/W\in[0,+\infty)\\ \Gamma_3: DR\in[0,+\infty) & \wedge \quad V/W=(V/W)_{sup}\\ \Gamma_4: DR=DR_{sup} & \wedge \quad V/W\in[0,+\infty)\end{cases}, \tag{1}$$

gde je Γ_1 globalni drift, ili horizontalni ugib, u oznaci *DR* (donja granica ili abscisa), Γ_2 relativna vrednost ukupne smičuće sile u osnovi objekta *V/W* (leva granica ili ordinata), Γ_3 gornja granica za supremum $(V/W)_{sup}$, Γ_4 desna granica za supremum DR_{sup}. Sa *V* označena je ukupna smičuća sila u osnovi objekta, a sa *W* ukupna težina. U slučaju odgovora zgrada u kapacitativnom domenu $CD_{NSPA,}$ primenom NSPA analize, za skup diskretnih uređenih parova DR_i i $(V/W)_i$ može se pisati:

where Γ_1 is a global drift, or horizontal deflection, denoted as *DR* (lower border or abscissa), Γ_2 the relative value of the total shear force in the base of structure *V/W* (left border or ordinate), Γ_3 upper border for the supremum $(V/W)_{sup}$, Γ_4 right border for the supremum DR_{sup}, *V* represents the total shear force in the base of the structure, while *W* is the total weight. In the case of the response of buildings in the capacity domain $CD_{NSPA,}$ by applying NSPA analysis, for the set of discrete and ordered pairs DR_i and $(V/W)_i$ it could be written:

$$\mathrm{CD}_{\mathrm{NSPA}}:\ \bigcup_{i=1}^{n}\langle DR_i,(V/W)_i\rangle, \qquad \forall (DR_i,(V/W)_i)\in\Re^+. \tag{2}$$



Slika 1. Odgovor 2D modela zgrada u kapacitativnom domenu
Figure 1. The response of 2D building models in a capacity domain

Model zgrade izlaže se dejstvu lateralnog seizmičkog opterećenja, a odgovor sistema prati se preko promene horizontalnog pomeranja najvišeg čvora zgrade. Inkrementalno-iterativna procedura izvršava se sve dok se ne dostigne unapred definisani nivo horizontalnog pomeranja D_{max} najvišeg čvora zgrade ili dok ne nastupi kolaps konstrukcije.

Definicija 1: *NSPA pushover kriva V/W=f(DR) jeste splajnom interpolirana kriva generisana povezivanjem diskretnih vrednosti iz inkrementalnih situacija* $I_i\langle DR_i,(V/W)_i\rangle$ *NSPA analize (slika 2):*

Model of building is exposed to the lateral seismic load and the response of the system is monitored through horizontal displacement of the highest building node. Incremental-iterative procedure is executed until a predefined level of horizontal displacement D_{max} of the highest node is reached, or until the collapse of the structure occurs.

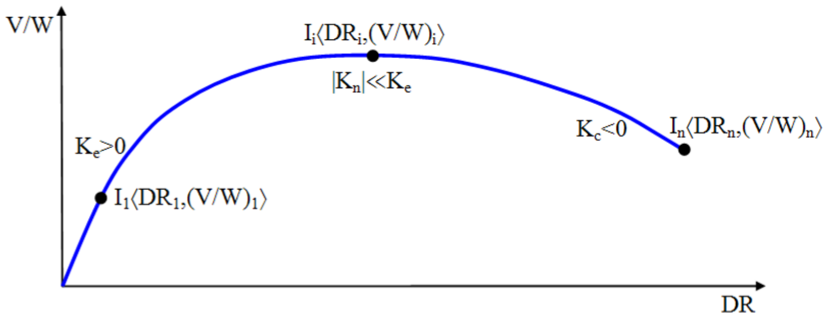
Definition 1: *NSPA pushover curve V/W=f(DR) is a curve interpolated by a spline generated by connecting discrete values from incremental situations* $I_i\langle DR_i,(V/W)_i\rangle$ *of NSPA analysis (Figure 2):*

$$V/W=f(DR), \tag{3}$$

pri čemu je:

where:

$$I_i\langle DR_i,(V/W)_i\rangle\subset[V/W=f(DR)], \qquad \forall (DR_i,(V/W)_i)\in\Re^+. \tag{4}$$



Slika 2. Pushover kriva V/W=f(DR)
Figure 2. Pushover curve V/W=f(DR)

Uspostavljanje kontinuiteta veze između diskretnih vrednosti $I_i\langle DR_i,(V/W)_i\rangle$ dobijenih NSPA analizom sprovodi se interpolacijom, gde se kao osnovni metod za uspostavljanje ove veze primenjuje linearna interpolacija. Mnogo bolje rešenje postiže se primenom interpolacije B-splajnovima, koji su generalizacija *Bezier*-ovih krivih [20]:

Establishing the continuity of the connection between discrete values $I_i\langle DR_i,(V/W)_i\rangle$, which are obtained using the NSPA analysis, is achieved by interpolation, where the linear interpolation represents the main method for establishing this relationship. Much better solution is achieved using the B-spline interpolation, as a generalization of *Bezier*'s curves [20]:

$$S(u)=\sum_{i=0}^{m} c_i N_i^n(u). \tag{5}$$

gde su *S(u)* B-splajn kriva stepena *n*. c_i kontrolne tačke. dok je B-splajn bazna funkcija:

where: *S(u)* represents a B-spline curve of a degree *n*. c_i control points. while the B-spline is a basis function:

$$N_i^0(u)=\begin{cases}1 & \text{za} \quad t_i \le u \le t_{i-1}\\ 0 & \end{cases}. \tag{6}$$

$$N_i^r(u)=\frac{u-t_i}{t_{i+r}-t_i}N_i^{r-1}(u)+\frac{t_{i+r+1}-u}{t_{i+r+1}-t_{i+1}}N_{i+1}^{r-1} \quad \text{za} \quad 1\le r\le n. \tag{7}$$

Reprezentativan model *pushover* krive karakterišu tri bitno različita domena: linearno-elastično ponašanje za koje je elastična krutost sistema pozitivna, K_e>0, nelinearno ponašanje za koje je nelinearna krutost sistema $|K_n| \ll K_e$ i kolaps koji karakteriše negativna krutost sistema K_c<0 i redukcija nosivosti. Tipologija *pushover* krivih, uvedena u ovom istraživanju, bazira se na analizi nelinearnog odgovora 3D modela zgrada, a takođe i odgovora određenog broja 2D višespratnih okvira [3], pošto oni participiraju kao konstruktivne celine zgrada. Kriterijumi na osnovu kojih se može sprovesti generalna podela *pushover* krivih jesu: globalna duktilnost *μ*, globalni driftovi za performansne nivoe armiranobetonskih okvirnih sistema (*structural performance levels*): DR_{IO} (IO − *immediate occupancy*), DR_{LS} (LS − *life safety*), DR_{CP} (CP − *collapse prevention*) i egzistencija linearnog (L − *linear*), nelinearnog (N − *nonlinear*) i kolapsnog (C − *collapse*) subdomena:

A representative model of pushover curve is characterized by the three very different domains: linear-elastic behaviour for which the elastic stiffness of the system is positive, K_e>0, the nonlinear behaviour for which the nonlinear system stiffness is $|K_n| \ll K_e$ and the collapse which is characterized by a negative stiffness K_c<0 and a strength reduction. The typology of pushover curves, introduced in this research, is based on an analysis of the nonlinear response of 3D building models, and also upon the response of a number of 2D multi-story frames [3], since they are participating as building's structural units. The criteria upon which the general classification of pushover curves may be done are: the global ductility *μ*, the global drifts for performance levels of the reinforced concrete frame structures: DR_{IO} (IO - immediate occupancy), DR_{LS} (LS - life safety), DR_{CP} (CP - collapse prevention) and the existence of the linear (L), a nonlinear (N) and a collapse (C) subdomain:

$$\mathrm{L}=\left[0, DR_y \pm \varepsilon_1\right]. \quad \mathrm{N}=\left[DR_y \pm \varepsilon_1, DR_c \pm \varepsilon_2\right]. \quad \mathrm{C}=\left[DR_c \pm \varepsilon_2, DR_{max}\right]. \tag{8}$$

U jednačini (8) DR_y jeste globalni drift za nivo granice tečenja, DR_c globalni drift za nivo iniciranja kolapsnog subdomena, DR_{max} globalni drift za nivo maksimalnih deformacija i ε_i su faktori korekcije za granične kriterijume. Veliki broj numeričkih testova, sprovedenih na okvirnim sistemima, ukazuje na varijaciju u odgovoru

Introduced in Eq.(8) DR_y is the global drift for the level of yield strength, DR_c global drift for the level of the collapse subdomain initiation, DR_{max} global drift for the level of maximum deformations and ε_i are the correction factors for borderline criteria. A large number of numerical tests, conducted upon the frame systems, *pushover* krivih pri nelinearnom ponašanju sistema.

Generalna podela zgrada prema nelinearnom odgovoru jeste na zgrade visoke duktilnosti (DCH − *high class ductility*), srednje duktilnosti (DCM − *medium class ductility*) i niske duktilnosti (DCL − *low class ductility*) [6]. Prvi slučaj nelinearnog odgovora karakteriše visoka duktilnost DCH, na osnovu čega su izvedene osobine krutosti sistema (slika 3) [2]:

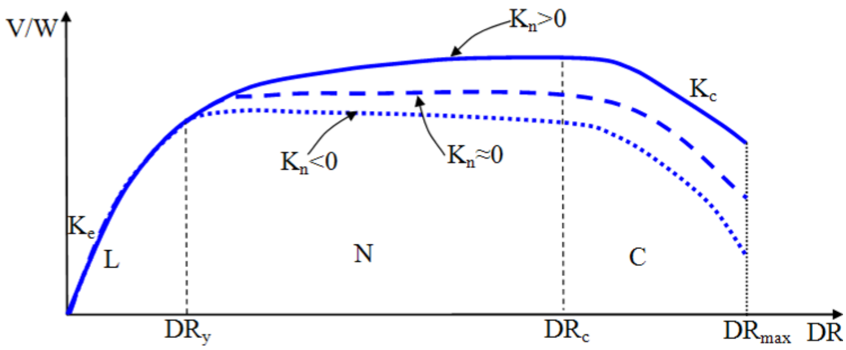
showed the variation in the responses of pushover curves during the nonlinear behaviour of the system.

The general classification of buildings according to nonlinear response has been made. Thus, there are a high ductility class (DCH), a medium ductility class (DCM) and a low ductility class (DCL) [6]. The first case of nonlinear response is characterized by the high ductility DCH, based on which the stiffness properties of the system are derived (Fig. 3) [2]:

$$\exists L,\ \exists N,\ \exists C,\quad K_e > 0,\ \ |K_n| < K_e,\ \ K_c < 0,\quad \forall (DR_i, (V/W)_i) \in \Re^+ . \tag{9}$$

Izražen N domen ukazuje na povoljno duktilno ponašanje zgrada s obzirom na mogućnost znatne disipacije histerezisne energije i plastifikaciju sistema povoljnim mehanizmima loma. Prema tome, moguće su sledeće varijante nelinearne krutosti sistema: K_n>0, K_n≈0 (K_n≪K_e) i K_n<0.

Prominent N domain points to a favourable ductile behaviour of buildings due to the possibility of substantial hysteretic energy dissipation and the plasticity of the system using favourable fracture mechanisms. Thus, the following variations of the nonlinear stiffness of the system are possible: K_n>0, K_n≈0 (K_n≪K_e) and K_n<0.



Slika 3. Odgovori sistema s duktilnim DCH ponašanjem ∃L, ∃N i ∃C
Figure 3. System responses with ductile DCH behaviour ∃L, ∃N and ∃C

Razmatranje nastanka kolapsa sistema kompleksan je problem i zahteva višekriterijumsku i višeparametarsku analizu, međutim može da se konstatuje da ukoliko je K_n>0, tada kolaps konstrukcije, u najvećem broju slučajeva, nastupa u C subdomenu. Ukoliko je K_n≈0, tada kolaps može nastupiti i u N i u C subdomenu, a ako je K_n<0, tada je iniciranje kolapsa, između ostalog, funkcija stepena redukcije nosivosti sistema i DR_{CP}, pri čemu će s velikom verovatnoćom nastupiti u N domenu.

Drugi slučaj nelinearnog odgovora karakteriše DCM i DCH duktilnost, pri čemu nema jasno izraženog C subdomena (slika 4):

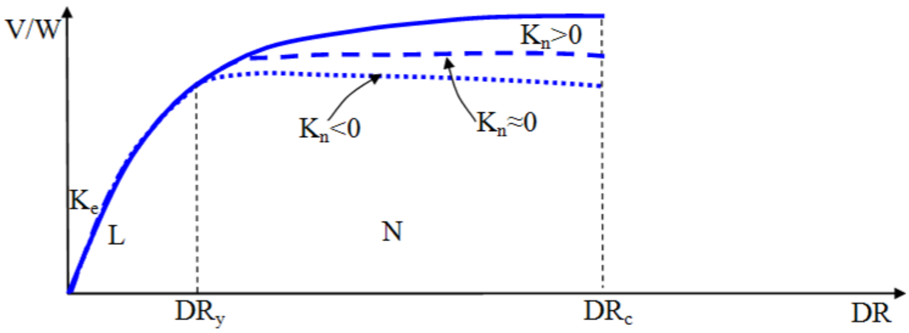
Consideration of the system collapse occurrence is a complex problem and requires a multiparametric analysis based on multiple criteria. However, it may be concluded that if K_n>0, then the collapse of structures, in most cases, occurs in the C subdomain. If K_n≈0, then the collapse may occur in the N and C subdomain, and if K_n<0, then the initiation of the collapse, among other things, depends on the level of the strength reduction of the system and the global drift DR_{CP}, while there is a high probability that it will occur in the N domain.

Another case of nonlinear response is characterized by DCM and DCH ductility, where the C subdomain is not clearly distinguished (Figure 4):

$$\exists L,\ \exists N,\quad K_e > 0,\ \ |K_n| < K_e,\ \ K_c = 0,\quad \forall (DR_i, (V/W)_i) \in \Re^+ . \tag{10}$$

Moguće varijante nelinearne krutosti sistema K_n iste su kao i za prethodni slučaj: K_n>0, K_n≈0 (K_n≪K_e) i K_n<0. Ovakav odgovor sistema karakteriše egzistencija L i N domena, a s druge strane, ovakav odgovor sistema može biti i problem numeričkog rešenja, pošto je kod određenih numeričkih modela potreban veliki broj inkremenata i iteracija, a potrebno je i pooštriti kriterijume tolerancije za rezidualno (neizbalansirano) opterećenje.

Possible variants of the nonlinear system stiffness K_n are the same as in the previous case: K_n>0, K_n≈0 (K_n≪K_e) and K_n<0. Such a system response is characterized by the existence of L and N domains, but on the other hand, this system response might be due to a problem with a numerical solution, since certain numerical models need a large number of increments and iterations, and also it is necessary to narrow the tolerance criteria for the residual (unbalanced) load.

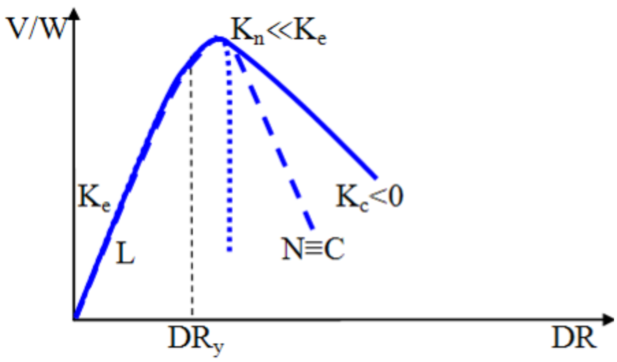


Slika 4. Odgovori sistema s duktilnim DCM i DCH ponašanjem ∃L, ∃N i ∄C
Figure 4. System responses with ductile DCM and DCH behaviour ∃L, ∃N and ∄C

Treći slučaj nelinearnog odgovora karakteriše nizak nivo duktilnosti DCL ili neduktilno to jest krto ponašanje, pri čemu nema jasno izraženog N i C subdomena (slika 5). U tom slučaju važi sledeće:

The third case of nonlinear response is characterized by a low level of ductility DCL or nonductile, i.e. brittle behavior, without the clearly expressed N and C subdomains (Figure 5). In that case the following applies:

$$\exists \mathrm{L}, \quad \exists \mathrm{N} \equiv \mathrm{C}, \quad K_e > 0, \quad K_n < 0, \quad K_c < 0, \quad \forall \left(DR_i, (V/W)_i \right) \in \Re^+ . \tag{11}$$



Slika 5. Odgovori sistema s niskim nivoom duktilnosti ∃L, ∃N (ili ∄N), ∃C (ili ∄C)
Figure 5. System responses with low ductility ∃L, ∃N (or ∄N), ∃C (or ∄C)

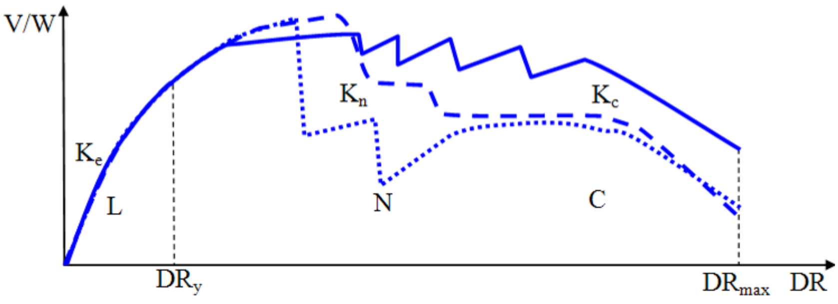
Prethodno su prezentovani generalizovani modeli nelinearnog ponašanja, međutim, u određenim situacijama, odgovor sistema može takođe pripadati i prelaznoj kategoriji. Karakterističan primer za to jeste nelinearni odgovor sistema s naglom promenom K_n u N domenu, tzv. oblik zubaca testere (*saw-tooth shape*) (slika 6) [10]:

Generalized models of nonlinear behaviour have been previously presented. However, in certain situations, the system's response may also belong to a transitional category. A characteristic example is a nonlinear response of the system with a sudden change of K_n in N domain (so called saw-tooth shape) (Figure 6) [10]:

$$\mathrm{N}: \quad K_n > 0, \quad K_n \approx 0, \quad K_n < 0 . \tag{12}$$

Sistem se generalno ponaša duktilno, dok je promena krutosti u N domenu frekventna. Prikazivanje *pushover* krive u tom slučaju može da se izvrši primenom kompatibilne krive kapaciteta.

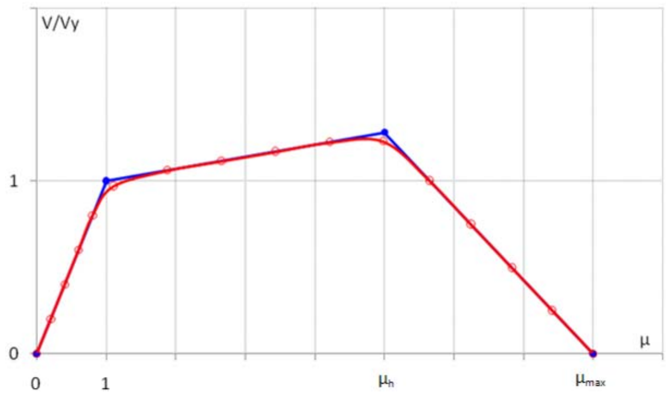
The system generally behaves in a ductile manner, while the change of stiffness in N domain is frequent. Presentation of the pushover curve in that case may be achieved by using the compatible capacity curve.



Slika 6. Odgovori sistema s duktilnim ponašanjem ∃L. ∃N. ∃C i promenljivim K_n
Figure 6. System responses with ductile behaviour ∃L, ∃N, ∃C and variable K_n

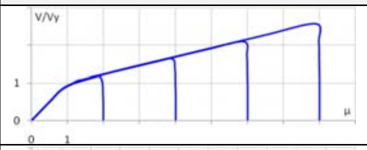
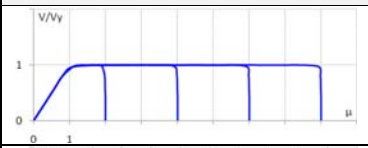
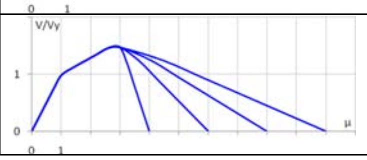
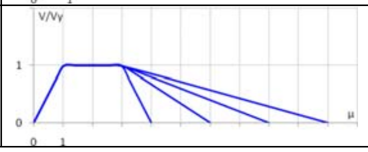
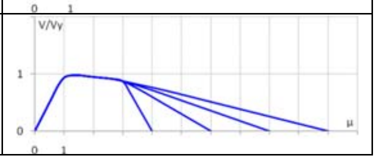
Klasifikacija tipova *pushover* krivih izvedena je na osnovu prethodno prikazanih osobina krutosti sistema i analizom varijacije parametara: μ duktilnosti, μ_h duktilnosti u zoni ojačanja/omekšanja i α koeficijenta odnosa krutosti u nelinearnom i linearnom domenu. Sve *pushover* krive prvo su konstruisane kao poligonalne linije na osnovu kojih su duž segmenata izabrane diskretne međuvrednosti, a zatim je sprovedena interpolacija splajnovima (slika 7). Dodatno je sprovedeno filtriranje diskretnih vrednosti za μ=1 i μ_h. U tabeli 1 prikazane su generisane *pushover* krive, pri čemu je na abscisi vrednost μ, a na ordinati normalizovana vrednost V/V_y. Razmatrane su *pushover* krive u funkciji varijacije parametara α>0, α≈0, α<0 i duktilnosti $\mu=\mu_h$ i $\mu>\mu_h$.

Classification of pushover curves is done according to the previously presented stiffness properties and upon the analysis of the variation of parameters: μ ductility, μ_h ductility in hardening/softening zone and α which represents the ratio of stiffnesses in the nonlinear and linear domains. All pushover curves were constructed as polygonal lines. Along the segments certain discrete intermediate values are selected and then the spline interpolation is conducted (Figure 7). Discrete values for μ=1 and μ_h have been filtered additionally. Table 1 shows the generated pushover curves, where value of μ is along the abscissa and the normalized value of V/V_y along the ordinate. The pushover curves were considered as a function of variation of parameters α>0, α≈0, α<0 and ductility $\mu=\mu_h$ and $\mu>\mu_h$.



Slika 7. Interpolirana pushover kriva s filtriranjem diskretnih vrednosti za μ=1 i μ_h
Figure 7. Interpolated pushover curve with filtering of discrete values for μ=1 and μ_h

Tabela 1. Generisane pushover krive u funkciji varijacije parametara μ, μ_h i α
Table 1. Generated pushover curves as a function of variation of parameters μ, μ_h and α

	α>0	α≈0	α<0
$\mu=\mu_h$			
$\mu>\mu_h$			

3 TIPOLOGIJA NSPA *PUSHOVER* POVRŠI

Razmatranje 3D seizmičkog odgovora zgrada moguće je sprovesti primenom *pushover* površi (*pushover surface*) za slučaj da se uzimaju u obzir dve istovremene ortogonalne komponente seizmičkog dejstva. Konstrukcija i generisanje *pushover* površi sprovodi se integracijom nelinearnih odgovora prikazanih preko individualnih *pushover* krivih koje su određene za različite uglove dejstva zemljotresa simultano u dva ortogonalna pravca [4]. Model zgrade izlaže se dejstvu lateralnog seizmičkog opterećenja čija je raspodela konstantna za konvencionalnu NSPA analizu (NSCPA − *Nonlinear Static Conventional Pushover Analysis*) u toku inkrementalnih proračunskih situacija. U slučaju adaptivne NSPA analize (NSAPA − *Nonlinear Static Adaptive Pushover Analysis*), raspodela lateralnog seizmičkog opterećenja je promenljiva u toku inkrementalnih opterećenja [18]. NSCPA i NSAPA analize sprovode se za različite vrednosti ugla θ_i, kojim je definisan pravac zemljotresa, u intervalu θ=[0,360°], po jedan slučaj opterećenja za svaki inkrement priraštaja ugla $\Delta\theta$. U fazama proračuna zgrade razmatraju se svi stepeni slobode, ali se monitoring odgovora sistema prati i prezentuje za odgovarajući pravac (ugao θ_i) i predstavlja *pushover* krivom za taj pravac. Integracijom ovako određenih *pushover* krivih $V/W=f(DR)_\theta$ po uglovima θ_i generiše se *pushover* površ 3D modela zgrade.

Odgovor 3D modela zgrada u kapacitativnom domenu prema NSPA analizi, a prezentovan primenom *pushover* krivih za uglove θ_i, u opštem slučaju definisan je granicama Γ_i (slika 8):

3 TYPOLOGY OF NSPA PUSHOVER SURFACES

Consideration of 3D seismic response of buildings is carried out by using a pushover surface in a case when two simultaneous orthogonal components of the seismic action are taken into account. Construction and generation of pushover surface is performed by integrating the nonlinear responses presened by the individual pushover curves determined for different angles of earthquake actions simultaneously in two orthogonal directions [4]. Building model is exposed to actions of lateral seismic load whose distribution is constant for conventional NSPA analysis (NSCPA - Nonlinear Static Conventional Pushover Analysis) during the incremental load cases. In the case of the adaptive NSPA analysis (NSAPA - Nonlinear Static Adaptive Pushover Analysis), distribution of lateral seismic load varies during the incremental loading [18]. NSCPA and NSAPA analyses are carried out for different values of the angle θ_i, which is defining direction of an earthquake, in the interval θ=[0,360°], one load case for each incremental angle $\Delta\theta$. In the stages of building analysis, all degrees of freedom are considered, but the monitoring of the system response is conducted for the corresponding direction (angle θ_i) and presented by the pushover curve for that direction. By integrating the set of so obtained pushover curves $V/W=f(DR)_\theta$ for angles $\theta_{i,}$ pushover surface of a 3D model of building is generated.

Response of 3D models of buildings in the capacity domain according to NSPA analysis, and presented using the pushover curves for the angles θ_i, in the general case is defined by the boundaries Γ_i (Figure 8):

$$\mathrm{CD}:\begin{cases}\Gamma_1:\left(DR_{x,i},DR_{y,i}\right)\in[0,+\infty) & \wedge \quad V/W=0\\ \Gamma_2:\left(DR_{x,i},DR_{y,i}\right)\in[0,+\infty) & \wedge \quad V/W=\left(V/W\right)_{sup}\\ \Gamma_3:\left(DR_{x,i},DR_{y,i}\right)=DR_{sup} & \wedge \quad V/W\in[0,+\infty)\end{cases}, \tag{13}$$

gde je Γ_1 donja granična ravan (baza) DR_x-DR_y za $V/W=0$, Γ_2 gornja granična ravan (baza) DR_x-DR_y za supremum $(V/W)_{sup}$, a Γ_3 granična površ za $DR_{x,sup}$ i $DR_{y,sup}$ (omotač). U slučaju odgovora NSPA analize zgrada u kapacitativnom domenu $\mathrm{CD_{NSPA}}$ za skup diskretnih uređenih parova DR_i i $(V/W)_i$ može se pisati:

where Γ_1 represents a bottom boundary plane (base) DR_x-DR_y for $V/W=0$, Γ_2 represents upper boundary plane (base) DR_x-DR_y for the supremum $(V/W)_{sup}$, while Γ_3 represents boundary surface for $DR_{x,sup}$ and $DR_{y,sup}$ (mantle). In the case of NSPA analyses of buildings' responses in the capacity domain $\mathrm{CD_{NSPA}}$ for a set of discrete ordered pairs DR_i and $(V/W)_i$ it may be written:

$$\mathrm{CD_{NSPA}}=\bigcup_{i=1}^{n}\left\langle DR_{x,i},DR_{y,i},\left(V/W\right)_i\right\rangle, \quad \forall\left(DR_{x,i},DR_{y,i},\left(V/W\right)_i\right)\in\Re^+, \tag{14}$$

dok odgovarajuća *pushover* kriva predstavlja interpolirane diskretne uređene parove $\langle DR_{x,i},DR_{y,i},(V/W)_i\rangle$:

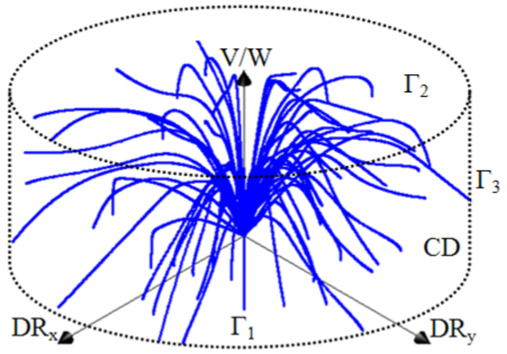
while the corresponding pushover curve represents the interpolated discrete ordered pairs $\langle DR_{x,i},DR_{y,i},(V/W)_i\rangle$:

$$V/W=f\left(DR_x,DR_y\right). \tag{15}$$

Na slici 10a prikazane su *pushover* krive generisane po uglovima θ_i u 2D ortogonalnom koordinatnom sistemu. dok su na slici 10b prikazane *pushover* krive u 3D ortogonalnom koordinatnom sistemu u izometriji. a generisane transformacijom:

Figure 10a shows the pushover curves generated for the angles θ_i in a 2D orthogonal coordinate system. while Figure 10b shows pushover curves in a 3D orthogonal coordinate system in isometry. generated by transformation:

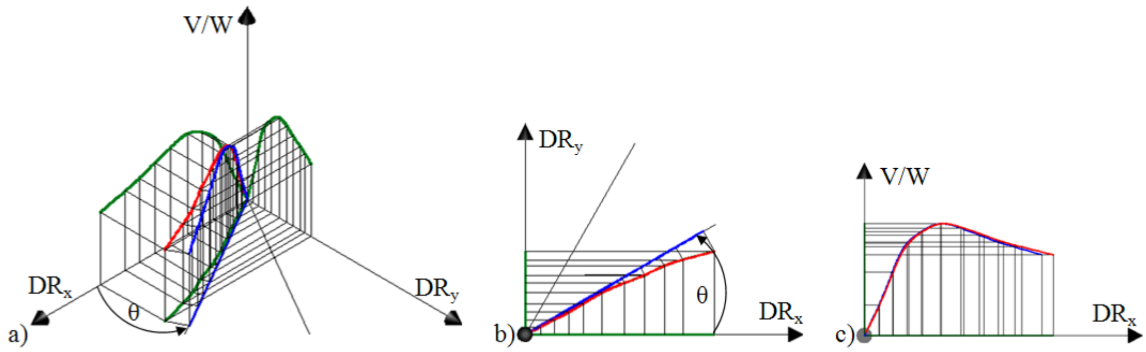
$$DR_x=DR_r\,cos\theta\,. \quad DR_y=DR_r\,sin\theta\,. \quad DR_r=\sqrt{DR_x^2+DR_y^2}\,. \tag{16}$$



Slika 8. Odgovor 3D modela zgrada u kapacitativnom domenu
Figure 8. The response of 3D models of buildings in the capacity domain

Konstrukcija *pushover* krive $V/W=f(DR)_\theta$ za ugao θ sprovodi se iz projekcije *pushover* krive sistema za bidirekciono seizmičko dejstvo na vertikalnu ravan DR_θ-V/W za ugao θ (slika 9a). Ortogonalne projekcije *pushover* krive za ugao θ prikazane su na slikama 9b i 9c za DR_x-DR_y ravan i DR_x-V/W ravan, respektivno.

Construction of pushover curve $V/W=f(DR)_\theta$ for the angle θ is carried out from the projection of pushover curve of the system for bidirectional seismic action on the vertical plane DR_θ-V/W for the angle θ (Figure 9a). Orthogonal projections of pushover curve for the angle θ are shown in Figures 9b and 9c for DR_x-DR_y and DR_x-V/W plane, respectively.



Slika 9. Pushover kriva u 3D koordinatnom sistemu za bidirekciono seizmičko dejstvo za ugao θ: a) projekcija na ortogonalne ravni DR_x-V/W, DR_y-V/W, DR_x-DR_y i za ugao θ; b) DR_x-DR_y ravan; c) DR_x-V/W ravan
Figure 9. Pushover curve in a 3D coordinate system for the bidirectional seismic action for the angle θ: a) projection on the orthogonal plane DR_x-V/W, DR_y-V/W, DR_x-DR_y and for an angle θ; b) DR_x-DR_y plane; c) DR_x-V/W plane

Definicija 2: *NSPA pushover površ 3D modela zgrade $V/W=f(DR_x,DR_y)$ jeste glatka interpolirana asimetrična rotaciona površ generisana povezivanjem diskretnih vrednosti iz inkrementalnih situacija $I_i\langle DR_i,(V/W)_{z,i},\theta_i\rangle$ NSPA analiza individualnih pushover krivih $(V/W)_i=f(DR_i,\theta_i)$ splajnovima u tangencijalnom pravcu $(V/W)_j=g(DR_{x,j},DR_{y,j})$ [4]:*

Definition 2: *NSPA pushover surface of a 3D model building $V/W=f(DR_x,DR_y)$ is an asymmetric smooth interpolated rotational surface generated by connecting discrete values obtained from incremental situations $I_i\langle DR_i,(V/W)_{z,i},\theta_i\rangle$ NSPA analysis of individual pushover curves $(V/W)_i=f(DR_i,\theta_i)$ using splines in the tangential direction $(V/W)_j=g(DR_{x,j},DR_{y,j})$ [4]:*

$$V/W = f(DR_x, DR_y) = \left[\bigcup_{\theta_i=0}^{360^\circ} f(DR_i, \theta_i)\right] \cup \left[\bigcup_{j=0}^{DR_{max}} g(DR_{x,j}, DR_{y,j})\right], \tag{17}$$

pri čemu je:

where:

$$I_i\langle DR_i,(V/W)_{z,i},\theta_i\rangle \subset [V/W = f(DR_x, DR_y)], \tag{18}$$

$$DR_{x,j}, DR_{y,j} \in [0, DR_{max}], \quad \theta_i \in [0, 360^\circ], \quad \forall (DR_i,(V/W)_i) \in \Re^+ . \tag{19}$$

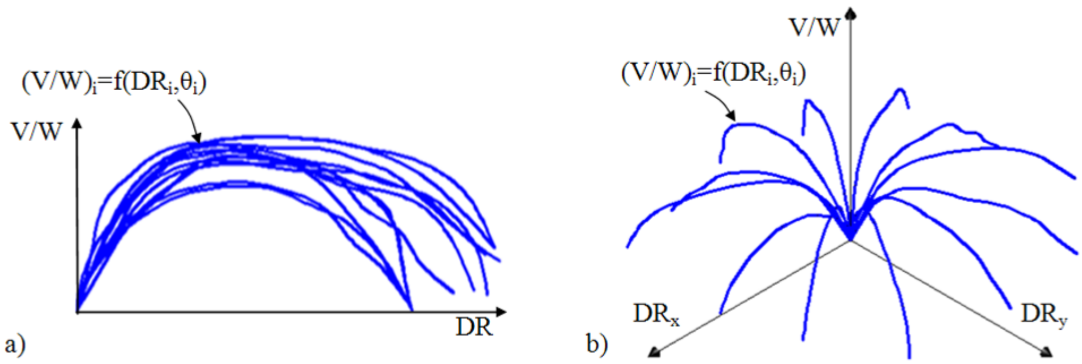
Na slici 10a prikazane su *pushover* krive generisane po uglovima θ_i u 2D ortogonalnom koordinatnom sistemu, dok su na slici 10b prikazane *pushover* krive u

Figure 10a shows the pushover curves generated for the angles θ_i in a 2D orthogonal coordinate system, while Figure 10b shows pushover curves in a 3D

3D ortogonalnom koordinatnom sistemu u izometriji, a generisane transformacijom:

orthogonal coordinate system in isometry, generated by transformation:

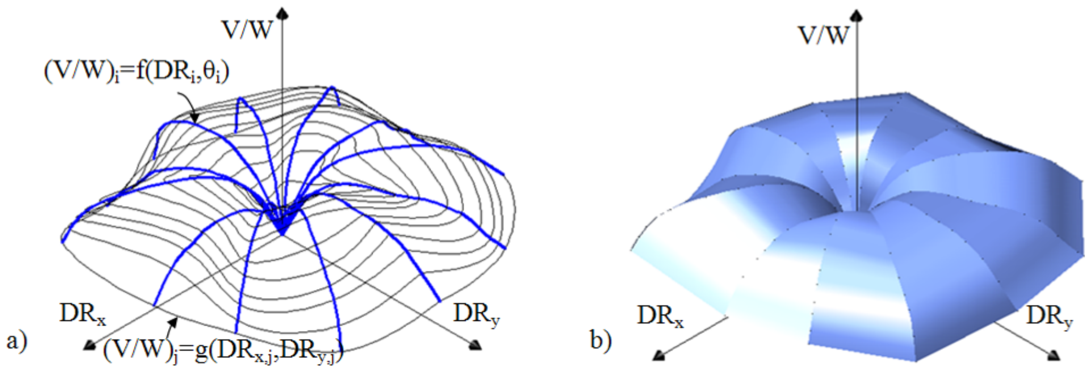
$$DR_x = DR_r \cos\theta, \quad DR_y = DR_r \sin\theta, \quad DR_r = \sqrt{DR_x^2 + DR_y^2}. \tag{20}$$



Slika 10. a) Pushover krive generisane za uglove θ_i u 2D ortogonalnom koordinatnom sistemu; b) NSPA pushover krive u 3D ortogonalnom koordinatnom sistemu u izometriji [4]
Figure 10. a) Pushover curves generated for angles θ_i in a 2D orthogonal coordinate system; b) NSPA pushover curves in a 3D orthogonal coordinate system in isometry [4]

3D mrežni model *pushover* krivih, povezanih splajnovima u tangencijalnom pravcu, prikazan je na slici 11a, dok je na slici 11b prikazan 3D model *pushover* površi u renderovanom prikazu.

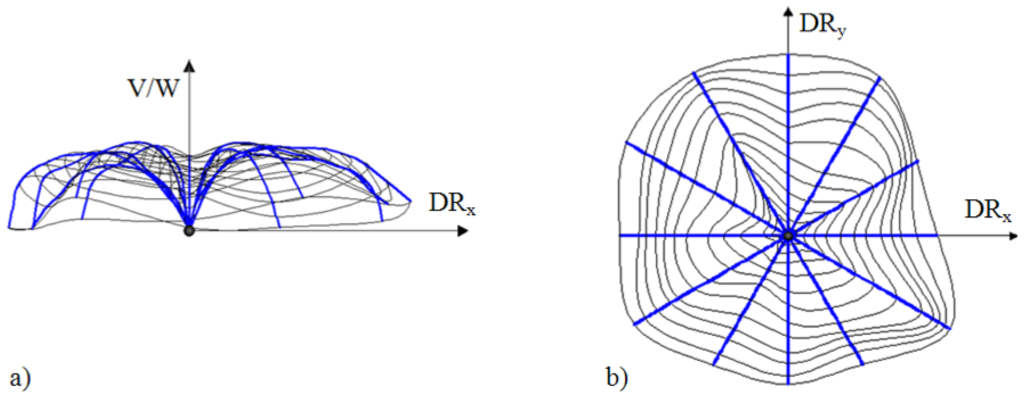
3D wireframe model of pushover curves, connected with splines in a tangential direction, is shown in Figure 11a, while Figure 11b shows a 3D model of the pushover surface in a rendered view.



Slika 11. a) 3D mrežni model pushover kriva; b) 3D renderovana pushover površ [4]
Figure 11. a) 3D wireframe model of pushover curves, b) 3D rendered pushover surface [4]

Prikazana *pushover* površ predstavlja opšti slučaj *pushover* površi za 3D model asimetrične neregularne zgrade i za analizirane uglove θ_i. Ortogonalne projekcije date *pushover* površi prikazane su na slici 12 za DR_x-V/W i DR_x-DR_y ravni. Splajnovi u tangencijalnom pravcu $(V/W)_j=g(DR_{x,j},DR_{y,j})$ povezuju diskretne vrednosti iz inkrementalnih situacija $I_i(DR_i,(V/W)_{z,i},\theta_i)$ NSPA analiza, pa, u opštem slučaju, nisu na ekvidistantnom odstojanju (slika 12a). S druge strane, ovi splajnovi, u opštem slučaju, nisu ni koncentrični krugovi (slika 12b).

Presented pushover surface represents the general case of a pushover surface for the 3D model of asymmetric and irregular building and analyzed angles θ_i. Orthogonal projections of given pushover surface are shown in the Figure 12 for the DR_x-V/W and DR_x-DR_y planes. Splines in the tangential direction $(V/W)_j=g(DR_{x,j},DR_{y,j})$ connect discrete values from incremental situations $I_i(DR_i,(V/W)_{z,i},\theta_i)$ of NSPA analyses and, in general, are not equidistant (Figure 12a). In addition, these splines, in general, are not concentric circles either (Figure 12b).



Slika 12. Ortogonalne projekcije za opšti slučaj pushover površi: a) DR_x-V/W, b) DR_x-DR_y ravan [4]
Figure 12. Orthogonal projections for the general case of pushover surface: a) DR_x-V/W, b) DR_x-DR_y plane [4]

Uspostavljanje kontinuiteta veze između diskretnih vrednosti $I_i\langle DR_i,(V/W)_i\rangle$, odnosno između *pushover* krivih $(V/W)_i=f(DR_i,\theta_i)$ u radijalnom pravcu i krivih $(V/W)_j=g(DR_{x,j},DR_{y,j})$ u tangencijalnom pravcu, sprovodi se primenom *Bezier*-ove površi [20]:

Establishing continuity of the connection between discrete values $I_i\langle DR_i,(V/W)_i\rangle$, or between the pushover curves $(V/W)_i=f(DR_i,\theta_i)$ in the radial direction and $(V/W)_j=g(DR_{x,j},DR_{y,j})$ curves in the tangential direction, is implemented using the *Bezier*'s surface [20]:

$$S(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_i^n(u)B_j^m(v)b_{i,j}\ , \tag{21}$$

gde je *Bernstein*-ov polinom:

Where the *Bernstein*'s polynomial is given by:

$$B_i^n(u)=\binom{n}{i}u^i(1-u)^{n-i}\ . \tag{22}$$

Kontrolni čvorovi definišu oblik NSPA *pushover* površi, dok vektor čvora određuje gde i kako *pushover* površ dodiruje kontrolne čvorove.

Analogija u geometrijskoj identifikaciji i matematičkoj prezentaciji *pushover* površi uspostavljena je sa polutorusnom površi kod koje je veći radijus torusa ekvivalentan manjem radijusu torusa *a* (*horn torus*) (slika 13a) [8]. Parametarske jednačine ovako generisane polutorusne površi glase:

Control nodes define the shape of pushover surface, while the vector of the node determines where and how the pushover surface touches the control nodes.

The analogy in geometrical identification and mathematical presentation of pushover surface may be established with a semi-torus surface where larger radius of torus is equivalent to smaller radius of the torus (horn torus) (Figure 13a) [8]. Parametric equations of the generated semi-torus surface are as follows:

$$x=(a+a\cos v)\cos u\ ,\quad y=(a+a\cos v)\sin u\ ,\quad z=a\sin v\ , \tag{23}$$

gde je centralni deo levkasta površ za koju važi:

where the central part is the funnel surface for which the following applies:

$$x=u\cos v\ ,\quad y=u\sin v\ ,\quad z=a\ln u\ . \tag{24}$$

Pushover površ jeste složena asimetrična površ koja se sastoji iz centralne površi i rotacione poligonalne površi, tako da svojom geometrijom asocira na polutorusnu površ, dok se u prirodi poređenje može uspostaviti s vulkanskim kraterom. Ukoliko se uzme u obzir to da je do nivoa iniciranja granice tečenja zavisnost sila – pomeranje linearna, tada je centralni deo *pushover* površi konusna površ za koju važi parametarska jednačina:

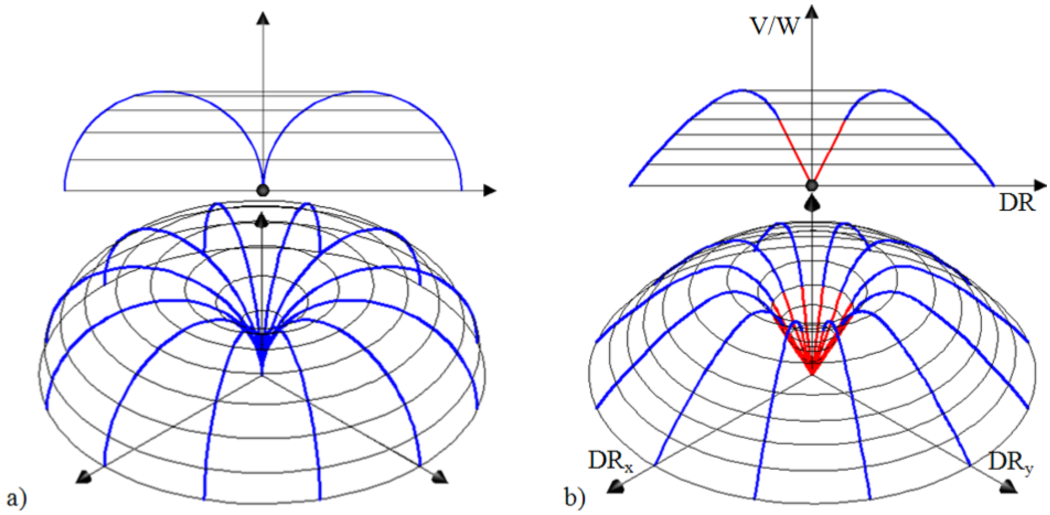
Pushover surface is a complex asymmetric surface consisting of the central surface and rotating polygonal surface, so its geometry resembles a semi-torus surface, while in the reality the association may be found with a volcanic crater. If one takes into account that up to the level of initiation of yield strength the force-displacement dependence is linear, then the central part of the pushover surface is a conical surface for which parametric equations may be applied:

$$x=\frac{h-u}{h}a\cos\theta\ ,\quad y=\frac{h-u}{h}a\sin\theta\ ,\quad z=u\ , \tag{25}$$

gde je *h* visina konusne površi (slika 13b). Na složenost *pushover* površi ukazuje i promenljivost *Gauss*-ove krivine u zavisnosti od razmatranog poddomena

where *h* is the height of the conical surface (Figure 13b). The variability of the *Gaussian* curvature depending on the considered subdomain of a pushover surface of a diskretnog modela *pushover* površi.

discrete model also indicates the complexity of the pushover surface.



Slika 13. Analogija u geometrijskoj identifikaciji i matematičkoj prezentaciji pushover površi: a) polutorusna površ sa R=a (horn torus); b) složena rotaciona pushover površ generisana iz konusne i poligonalne površi
Figure 13. The analogy in geometrical identification and mathematical presentation of pushover surface: a) semi-torus surface with R=a (horn torus), b) a complex rotational pushover surface generated from the conical and polygonal surfaces

Mnoga softverska rešenja primenjuju interpolaciju površi za određene matematičke funkcije i mapiranje, dok manji broj njih ima mogućnost prezentacije zakrivljene površi za diskretne vrednosti koordinata tačaka koje nisu uniformno raspodeljene. Problem koji se može pojaviti kod interpolacije *pushover* površi u ovom slučaju jeste odstupanje od realistične prezentacije, tako što se u određenim slučajevima diskretne vrednosti ne povezuju direktno interpolacionim funkcijama. Najbolje rešenje dobija se kada se diskretne vrednosti linearno interpoliraju i direktno povezuju u prostornom koordinatnom sistemu. Na taj način, generisana *pushover* površ formira se iz niza četvorouglova (trapeza), dok se dodatna rafiniranost može postići diskretizacijom trouglovima.

Tipološka analiza *pushover* površi znatno je kompleksnija od tipologije *pushover* krivih, pošto u jednoj *pushover* površi učestvuje više *pushover* krivih. Generalno, tipologija se može izvršiti s podelom površi na dve bitno različite grupe [4]: sa konstantnim znakom krutosti K_n u nelinearnom domenu za sve *pushover* krive i sa promenljivim znakom krutosti K_n u nelinearnom domenu za *pushover* krive. U grupi sa konstantnim znakom K_n moguće su varijacije: $K_n>0$ za sve *pushover* krive, $K_n≈0$ za sve *pushover* krive i $K_n<0$ za sve *pushover* krive, uz značajno učešće *P-Δ* efekata. U grupi sa promenljivim znakom K_n moguća je situacija da određene *pushover* krive imaju $K_n>0$, a da neke druge krive imaju $K_n<0$. Ovakav odgovor sistema jeste posledica znatne razlike tangentne krutosti zgrade za dva ortogonalna pravca. Ukoliko se uzme u obzir ispitivanje egzistencije linearnog, nelinearnog i kolapsnog subdomena, tada postoji još veći broj varijacija *pushover* površi.

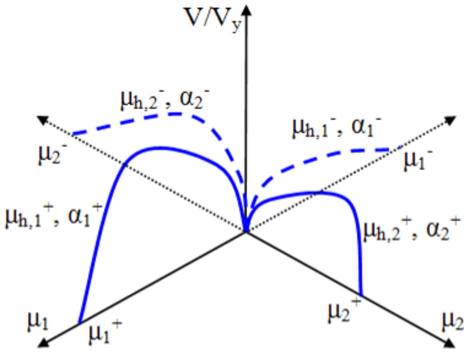
Generalizacija u tipologiji *pushover* površi izvedena je u funkciji varijacije μ, μ_h i α parametara za jedan glavni

A large number of software solutions use the surface interpolation for the given mathematical functions and mappings, while only few of them have the possibility to present a curved surface for discrete values of point coordinates that are not uniformly distributed. A problem that can occur during the pushover surface interpolation in this case is a deviation from the realistic presentation, so that in certain cases the discrete values are not directly connected by the interpolating functions. The best solution is obtained when the discrete values are linearly interpolated and directly linked in the spatial coordinate system. The pushover surface generated in this way is formed by a series of trapezoids, while an additional refinement may be achieved by discretization using triangles.

Typological analysis of the pushover surface is considerably more complex than the typology of pushover curves, since in one pushover surface more pushover curves participate. Overall, the typology may be made by classification of surfaces into the two significantly different groups [4]: with a constant sign of stiffness K_n in the nonlinear domain for all pushover curves and with a variable signs of stiffness in the nonlinear domain K_n for pushover curves. In the group with a constant sign K_n variations are possible: $K_n>0$ for all pushover curves, $K_n≈0$ for all pushover curves and $K_n<0$ for all pushover curves, with a significant participation of *P-Δ* effects. In the group with a variable sign of K_n for pushover curves it is possible that certain pushover curves have $K_n>0$, while some other curves have $K_n<0$. Such a response of the system is the result of significant differences in the tangent stiffness of the building in two orthogonal directions. If one takes into account the existence of linear, nonlinear and collapse subdomains, then there is an even larger variation of pushover surfaces.

pravac i za dva glavna pravca (slika 14). U slučaju varijacije μ, μ_h i α parametara kod *pushover* površi samo za jedan glavni pravac, razmatrani su slučajevi: identični parametri po smerovima jednog (svih) pravca i različiti parametri po smerovima jednog pravca.

Generalization in the typology of pushover surface is derived as a function of variation of parameters μ, μ_h and α for one and for two principal directions (Figure 14). In the case of variations of parameters μ, μ_h and α of the pushover surface only for one main direction, the following cases have been considered: identical parameters in the senses of one (all) direction and different parameters in senses of one direction.

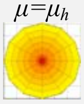
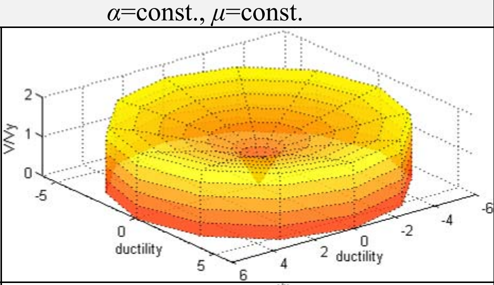
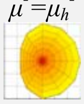
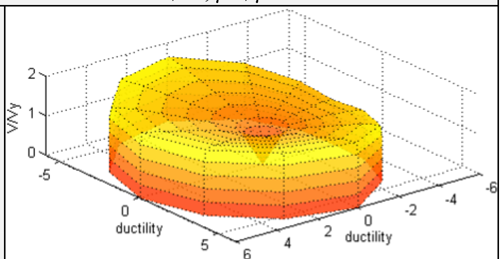
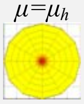
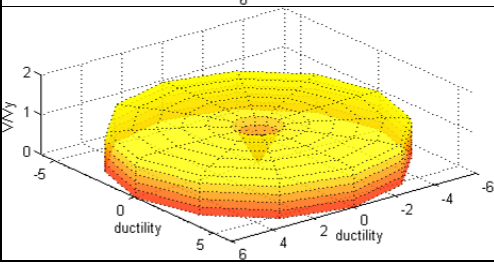
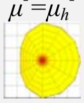
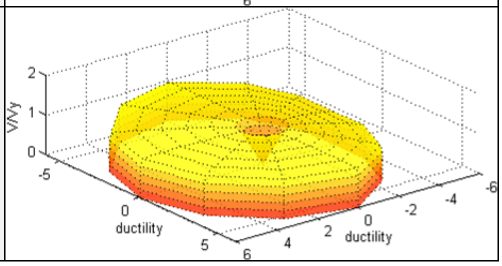


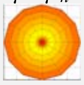
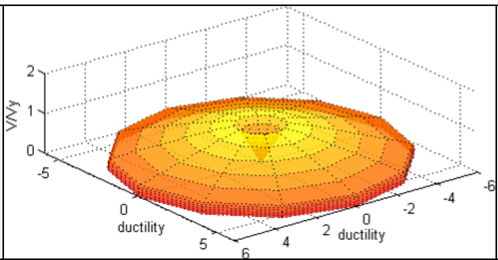
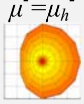
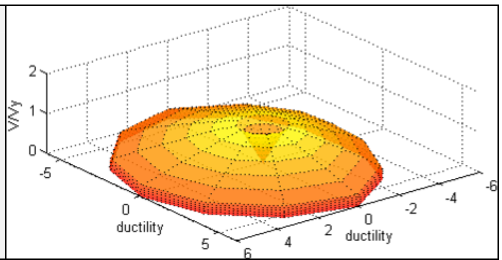
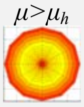
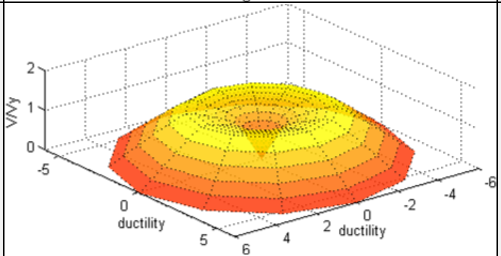
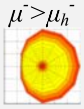
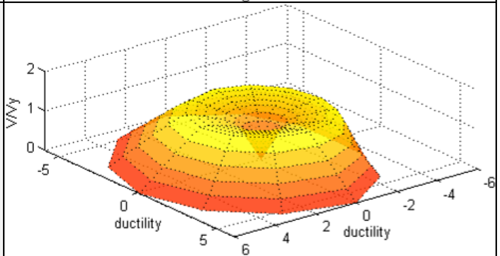
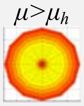
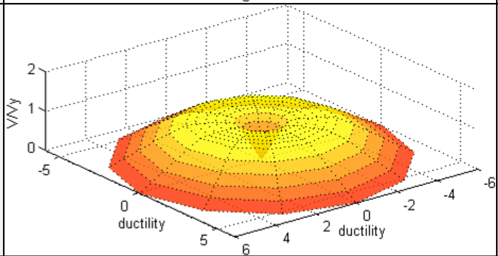
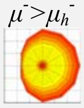
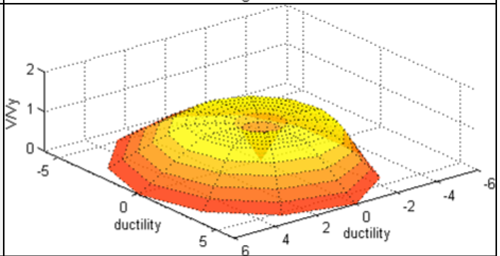
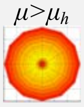
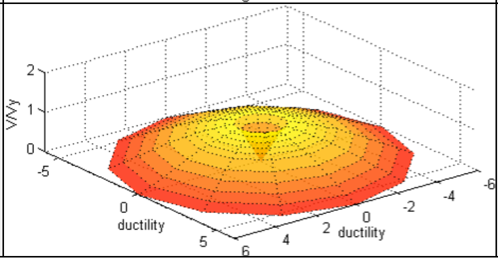
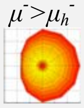
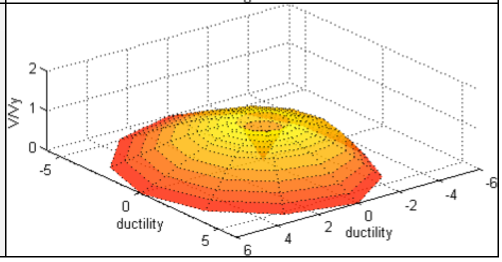
Slika 14. Generalizacija u tipologiji pushover površi u funkciji varijacije parametara μ, μ_h i α
Figure 14. Generalization in the typology of pushover surface as a function of variation of parameters μ, μ_h and α

Pushover površi generisane su primenom softvera MATLAB [17], gde su diskretne vrednosti linearno interpolirane i direktno povezane u prostornom koordinatnom sistemu. U tabeli 2 prikazane su generisane *pushover* površi u funkciji varijacije parametra $\alpha>0$, $\alpha\approx0$, $\alpha<0$ i duktilnost $\mu=\mu_h$ i $\mu>\mu_h$ za jedan glavni pravac.

Pushover surfaces were generated using the MATLAB software [17], where the discrete values were linearly interpolated and directly connected in the spatial coordinate system. Table 2 shows the generated pushover surface as the function of the variation of parameters $\alpha>0$, $\alpha\approx0$, $\alpha<0$ and the ductility $\mu=\mu_h$ and $\mu>\mu_h$ for one main direction.

Tabela 2. Generisane pushover površi u funkciji varijacije parametara μ, μ_h i α za jedan glavni pravac
Table 2. Pushover surfaces generated as a function of variation of parameters μ, μ_h and α for one main direction

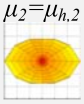
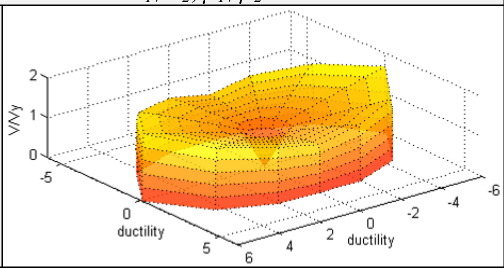
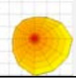
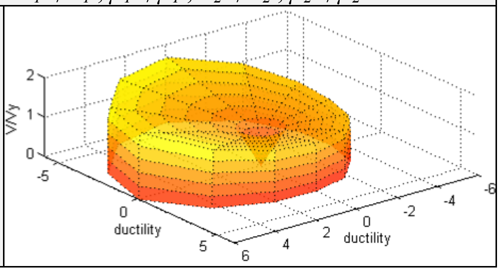
identične vrednosti parametara po smerovima jednog (svih) pravca identical values of parameters for the senses of one (all) direction α=const., μ=const.		različite vrednosti parametara po smerovima jednog pravca different values of parameters for the senses of one direction $\alpha^+\neq\alpha^-$, $\mu^+\neq\mu^-$	
$\alpha>0$ $\mu=\mu_h$ 		$\alpha^+>0$ $\mu^+=\mu_h^+$ $\alpha^->0$ $\mu^-=\mu_h^-$ 	
$\alpha\approx0$ $\mu=\mu_h$ 		$\alpha^+\approx0$ $\mu^+=\mu_h^+$ $\alpha^-\neq0$ $\mu^-=\mu_h^-$ 	

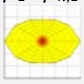
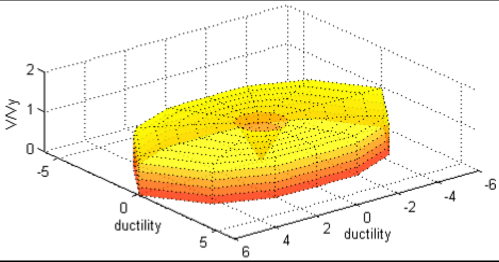
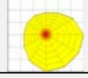
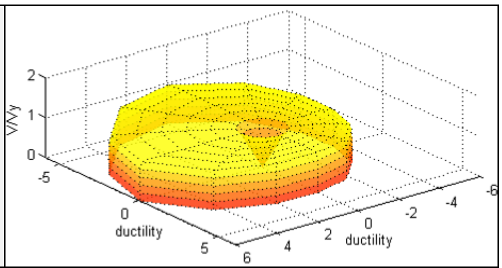
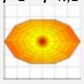
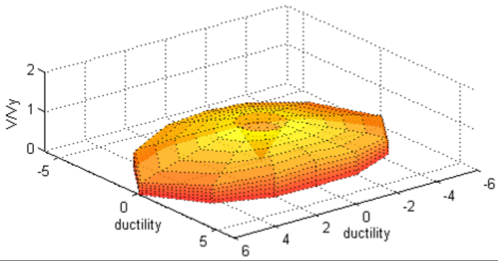
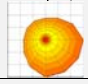
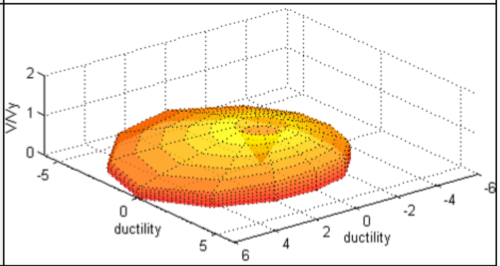
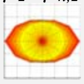
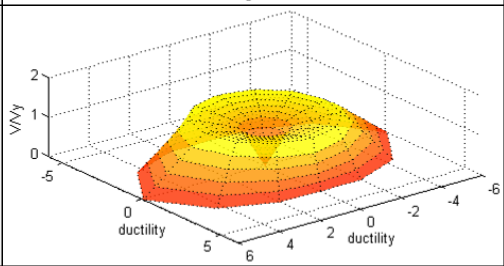
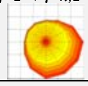
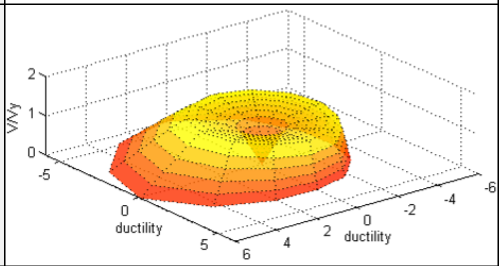
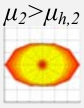
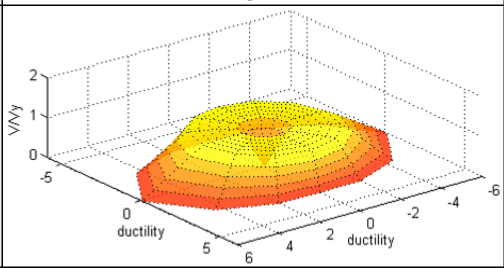
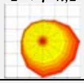
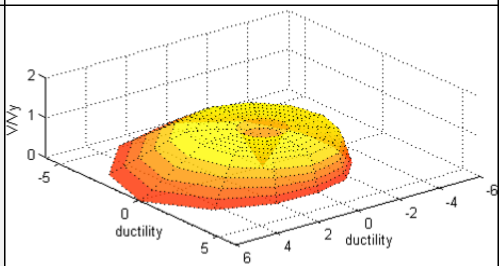
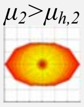
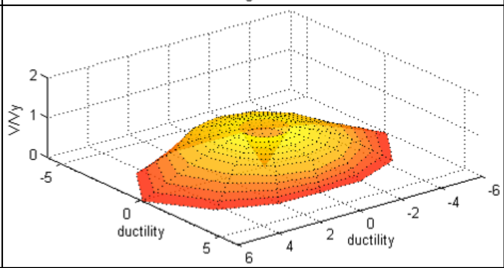
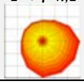
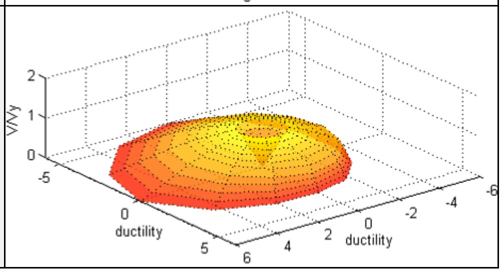
$\alpha<0$ $\mu=\mu_h$ 		$\alpha^+<0$ $\mu^+=\mu_h^+$ $\alpha^-<0$ $\mu^-=\mu_h^-$ 	
$\alpha>0$ $\mu>\mu_h$ 		$\alpha^+>0$ $\mu^+>\mu_h^+$ $\alpha^->0$ $\mu^->\mu_h^-$ 	
$\alpha\approx0$ $\mu>\mu_h$ 		$\alpha^+\approx0$ $\mu^+>\mu_h^+$ $\alpha^-\neq0$ $\mu^->\mu_h^-$ 	
$\alpha<0$ $\mu>\mu_h$ 		$\alpha^+<0$ $\mu^+>\mu_h^+$ $\alpha^-<0$ $\mu^->\mu_h^-$ 	

U slučaju varijacije parametara $\alpha>0$, $\alpha\approx0$, $\alpha<0$ i duktilnosti $\mu=\mu_h$ i $\mu>\mu_h$ kod *pushover* površi za dva glavna pravca, razmatrani su slučajevi: identični parametri po smerovima jednog pravca i različiti parametri po smerovima jednog pravca. U tabeli 3 su prikazane generisane *pushover* površi u funkciji varijacije parametra μ, μ_h i α za dva glavna pravca.

In the case of variation of parameters $\alpha>0$, $\alpha\approx0$, $\alpha<0$ and ductility $\mu=\mu_h$ and $\mu>\mu_h$ of the pushover surface in the two main directions, the following cases have been considered: identical parameters along the senses of one direction and different parameters along the senses of one direction. Table 3 shows the generated pushover surfaces as a function of variation of parameters μ, μ_h and α in the two main directions.

Tabela 3. Generisane pushover površi u funkciji varijacije parametara μ, μ_h i α za dva glavna pravca
Table 3. Generated pushover surfaces as a function of variation of parameters μ, μ_h and α for the two main directions

identične vrednosti parametara po smerovima jednog pravca identical values of parameters for the senses of one direction $\alpha_1\neq\alpha_2$, $\mu_1\neq\mu_2$		različite vrednosti parametara po smerovima jednog pravca different values of parameters for the senses of one direction $\alpha_1^+\neq\alpha_1^-$, $\mu_1^+\neq\mu_1^-$, $\alpha_2^+\neq\alpha_2^-$, $\mu_2^+\neq\mu_2^-$	
$\alpha_1>0$ $\mu_1=\mu_{h,1}$ $\alpha_2>0$ $\mu_2=\mu_{h,2}$ 		$\alpha_1>0$ $\alpha_1^+\neq\alpha_1^-$ $\mu_1^\pm=\mu_{h,1}^\pm$ $\alpha_2>0$ $\alpha_2^+\neq\alpha_2^-$ $\mu_2^\pm=\mu_{h,2}^\pm$ 	

$\alpha_1 \approx 0$ $\mu_1 = \mu_{h,1}$ $\alpha_2 \approx 0$ $\mu_2 = \mu_{h,2}$ 		$\alpha_1 \approx 0$ $\alpha_1^+ \neq \alpha_1^-$ $\mu_1^\pm = \mu_{h,1}^\pm$ $\alpha_2 \neq 0$ $\alpha_2^+ \neq \alpha_2^-$ $\mu_2^\pm = \mu_{h,2}^\pm$ 	
$\alpha_1 < 0$ $\mu_1 = \mu_{h,1}$ $\alpha_2 < 0$ $\mu_2 = \mu_{h,2}$ 		$\alpha_1 < 0$ $\alpha_1^+ \neq \alpha_1^-$ $\mu_1^\pm = \mu_{h,1}^\pm$ $\alpha_2 < 0$ $\alpha_2^+ \neq \alpha_2^-$ $\mu_2^\pm = \mu_{h,2}^\pm$ 	
$\alpha_1 > 0$ $\mu_1 > \mu_{h,1}$ $\alpha_2 > 0$ $\mu_2 > \mu_{h,2}$ 		$\alpha_1 > 0$ $\alpha_1^+ \neq \alpha_1^-$ $\mu_1^\pm \neq \mu_{h,1}^\pm$ $\alpha_2 > 0$ $\alpha_2^+ \neq \alpha_2^-$ $\mu_2^\pm \neq \mu_{h,2}^\pm$ 	
$\alpha_1 \approx 0$ $\mu_1 > \mu_{h,1}$ $\alpha_2 \approx 0$ $\mu_2 > \mu_{h,2}$ 		$\alpha_1 \approx 0$ $\alpha_1^+ \neq \alpha_1^-$ $\mu_1^\pm \neq \mu_{h,1}^\pm$ $\alpha_2 \neq 0$ $\alpha_2^+ \neq \alpha_2^-$ $\mu_2^\pm \neq \mu_{h,2}^\pm$ 	
$\alpha_1 < 0$ $\mu_1 > \mu_{h,1}$ $\alpha_2 < 0$ $\mu_2 > \mu_{h,2}$ 		$\alpha_1 < 0$ $\alpha_1^+ \neq \alpha_1^-$ $\mu_1^\pm \neq \mu_{h,1}^\pm$ $\alpha_2 < 0$ $\alpha_2^+ \neq \alpha_2^-$ $\mu_2^\pm \neq \mu_{h,2}^\pm$ 	

Komparacijom generisanih *pushover* površi, prikazanih u tabelama 2 i 3, mogu da se identifikuju četiri bitno različite grupe tih površi. U prvu grupu svrstavaju se *pushover* površi rotaciono polisimetrične u osnovi (leva strana tabele 2), gde su sve *pushover* krive identične, a splajnovi u tangencijalnom pravcu su koncentrični krugovi s centrom u vertikalnoj osi:

By comparing the generated pushover surfaces shown in Tables 2 and 3, four distinctly different groups of surfaces may be identified. The first group includes pushover surfaces rotationally poly-symmetric at their base (left side of Table 2), where all pushover curves are identical, and splines in tangential direction are the concentric circles with the center along the vertical axis:

$$\left(V/V_y\right)_{\theta 1,\max} = \left(V/V_y\right)_{\theta 2,\max} = \ldots = \left(V/V_y\right)_{\theta i,\max} = \ldots = \left(V/V_y\right)_{\theta n,\max}, \tag{26}$$

$$\mu_x^2 + \mu_y^2 = \mu_{max}^2, \tag{27}$$

$$\mu_y = \pm\sqrt{\mu_{max}^2 - \mu_x^2}, \quad \mu_x \in \left[-\mu_{max}, \mu_{max}\right], \tag{28}$$

gde je μ_{max} maksimalna realizovana duktilnost. Posledica ovog pravila jeste to što su svi splajnovi u tangencijalnom pravcu paralelni:

where μ_{max} is maximum realized ductility. The consequence of this rule is that all the splines in the tangential direction are parallel:

$$\left(V/V_y\right)_{j=1}=g\left(\mu_{x,j=1},\mu_{y,j=1}\right)\|\ldots\|\left(V/V_y\right)_{j=n}=g\left(\mu_{x,j=n},\mu_{y,j=n}\right). \quad (29)$$

U drugu grupu svrstavaju se *pushover* površi koje su monosimetrične u osnovi (desna strana tabele 2) i za koje linija u osnovi može da se prikaže kao zatvorena ovalna kriva [9]. Izraz za zatvorenu ovalnu krivu dodatno je korigovan i prilagođen za potrebe ovog istraživanja, tako da glasi:

The second group includes pushover surfaces which are mono-symmetric at their base (right side of Table 2) and for which the function of the closed oval curve may be used to represent their base [9]. The function of the closed oval curve is additionally corrected and adopted for this research, so the final form of this function is:

$$\left(\mu_x^2+p^2\mu_y^2\right)^2=\left(\mu_{x,max}+\left|\mu_{x,min}\right|\right)\mu_x^2+2\left(\mu_{x,max}+\left|\mu_{x,min}\right|\right)p^2\mu_x\mu_y^2, \quad (30)$$

$$\mu_y=\pm\frac{\sqrt{\mu_x\left(\mu_{x,max}+\left|\mu_{x,min}\right|-\mu_x+\sqrt{\left(\mu_{x,max}+\left|\mu_{x,min}\right|\right)\left(\mu_{x,max}+\left|\mu_{x,min}\right|-\mu_x\right)}\right)}}{p},\ldots\ \mu_x\in\left[\mu_{x,min},\mu_{x,max}\right], \quad (31)$$

gde je $\mu_{x,max}$ maksimalna realizovana duktilnost za *x* pravac, $\mu_{x,min}$ minimalna realizovana duktilnost za *x* pravac, a *p* parametar ovalne krive. Treću grupu čine *pushover* površi koje su bisimetrične u osnovi (leva strana tabele 3) i za koje linija u osnovi može da se prikaže kao superelipsa [9]. Izraz za superelipsu dodatno je korigovan i prilagođen za potrebe ovog istraživanja, tako da glasi:

where $\mu_{x,max}$ is maximum realized ductility for *x* direction, $\mu_{x,min}$ is minimum realized ductility for *x* direction, and *p* parameter of the oval curve. The third group represents the pushover surfaces which are bi-symmetric at their base (left side of Table 3) and for which the function of the superellipse may be used to represent their base [9]. The function of the superellipse is additionally corrected and adopted for this research, so the final form of this function is:

$$\left(\frac{\mu_x}{\mu_{x,max}}\right)^n+\left(\frac{\mu_y}{\mu_{y,max}}\right)^n=1, \quad (32)$$

$$\mu_y=\pm\mu_{y,max}\sqrt[n]{1-\left(\frac{\mu_x}{\mu_{x,max}}\right)^n}, \qquad \mu_x\in\left[-\mu_{x,max},\mu_{x,max}\right], \quad (33)$$

gde je $\mu_{y,max}$ maksimalna realizovana duktilnost za *y* pravac, dok je *n* parametar superelipse. U četvrtu grupu svrstavaju se *pushover* površi koje su asimetrične ili monosimetrične u osnovi (desna strana tabele 3) i za koje linija u osnovi može da se prikaže takođe kao zatvorena ovalna kriva. U odnosu na izraz (30), dodatno je sprovedena korekcija, tako da sada taj izraz glasi:

where $\mu_{y,max}$ is maximum realized ductility for *y* direction, while *n* is parameter of the superellipse. The fourth group includes pushover surfaces which are asymmetric or monosymmetric in their base (right side of Table 3) and for which the closed oval curve may be used to represent their base. Additional corrections were done for equation (30), so the expression is now:

$$\mu_{x,r}=\mu_{x0}+\mu_x\cos\varphi-\mu_y\sin\varphi, \qquad \mu_{y,r}=\mu_{y0}+\mu_x\sin\varphi+\mu_y\cos\varphi, \quad (34)$$

gde su: μ_x i μ_y duktilnosti prema izrazu (30), $\mu_{x,0}$ i $\mu_{x,0}$ duktilnosti za centar zatvorene ovalne krive, a φ je ugao rotacije zatvorene ovalne krive. U tabeli 4 prikazana su četiri karakteristična tipa *pushover* površi i konstruisane odgovarajuće krive u osnovi.

Specijalni slučaj *pushover* površi javlja se kada se primeni adaptivna NSAPA-DBA (*displacement based analysis*) analiza sa identičnim maksimalnim realizovanim pomeranjima po svim *pushover* krivama kod 3D modela zgrada:

where μ_x and μ_y are ductilities according to (30), $\mu_{x,0}$ and $\mu_{x,0}$ ductilities for the center of closed oval curve, while φ is the angle of rotation of the closed oval curve. Table 4 shows the four distinctly different groups of pushover surfaces and corresponding curves at their base.

Special case of the pushover surface occurs when applying adaptive NSAPA-DBA analysis (*displacement based analysis*) with the identical maximum achieved displacements for all pushover curves in a 3D model of a building:

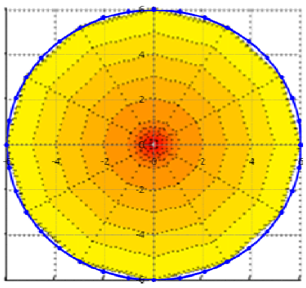
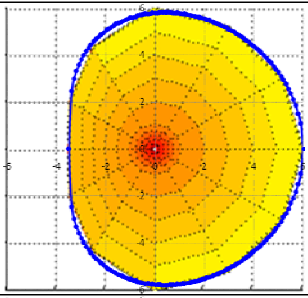
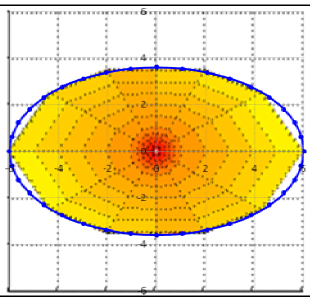
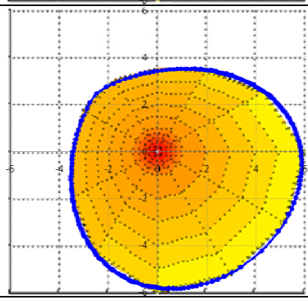
$$DR_{\theta 1,\max}=DR_{\theta 2,\max}=\ldots=DR_{\theta i,\max}=\ldots=DR_{\theta n,\max}. \quad (35)$$

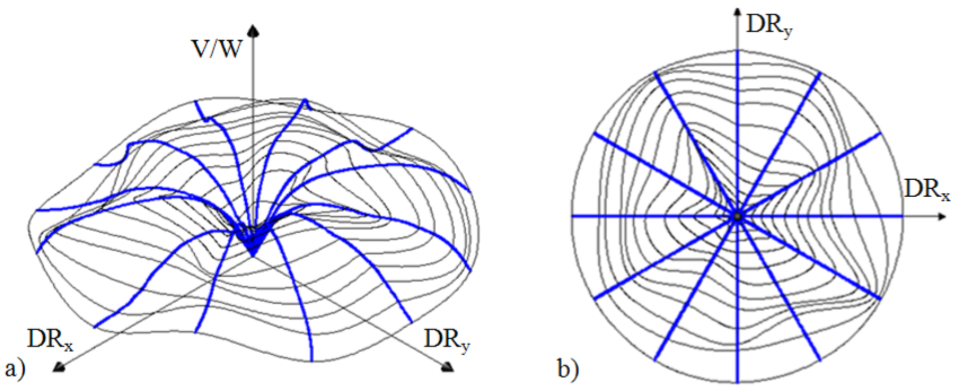
Tada *pushover* površ ima izgled prikazan na slici 15. Splajn u tangencijalnom pravcu za maksimalna realizovana pomeranja u ortogonalnoj projekciji DR_x-DR_y jeste krug (slika 15b), dok unutrašnji splajnovi u tangencijalnom pravcu ne moraju biti koncentrični krugovi. Iako su realizovana identična maksimalna pomeranja po svim *pushover* krivama, odgovarajuće vrednosti *V/W* nisu identične:

Then the pushover surface looks like the one shown in Figure 15. Spline in the tangential direction for the maximum achieved displacements in the orthogonal projection DR_x-DR_y is the circle (Figure 15b), while the internal splines in the tangential direction are not concentric circles. Even though the maximum realized displacements are identical along all pushover curves, the corresponding values of *V/W* are not identical:

$$\left(V/W\right)_{\theta 1,\max} \neq \left(V/W\right)_{\theta 2,\max} \neq \ldots \neq \left(V/W\right)_{\theta i,\max} \neq \ldots \neq \left(V/W\right)_{\theta n,\max} . \quad (35)$$

Tabela 4. Četiri karakteristična tipa pushover površi i konstruisane odgovarajuće krive u osnovi
Table 4. Four characteristic types of pushover surfaces and adequate curves at their base

μ_{max}=6		$\mu_{x,max}$=6 $\mu_{x,min}$=-3.5 p=1.3	
$\mu_{x,max}$=6 $\mu_{y,min}$=-3.5 n=2		$\mu_{x,max}$=6 $\mu_{x,min}$=-3.5 p=1.65 μ_{x0}=1.75 μ_{y0}=-1.75 φ=135°	



Slika 15. Pushover površ za slučaj da su maksimalna realizovana pomeranja identična po svim pushover krivama: a) izometrija; b) DR_x-DR_y ravan
Figure 15. Pushover surface in the case when the maximum realized displacements are identical for all pushover curves: a) isometry, b) DR_x-DR_y plane

4 ZAKLJUČAK

Analiza seizmičkog odgovora konstrukcija primenom *pushover* površi predstavlja originalno razvijeno rešenje autora ovog rada. Primenom *pushover* površi moguće je znatno kompleksnije i kompletnije razmotriti odgovor i performanse 3D modela konstrukcija, posebno u slučaju izražene nesimetrije mase i krutosti, izloženih bidirekcionom seizmičkom dejstvu. Postavka od koje se pošlo u razvoju matematičke formulacije i generisanju *pushover* površi jeste primena *pushover* krive za odgovor sistema u jednom pravcu. Integracijom odgovora sistema za veći broj pravaca, odnosno uglova dejstva, prikazuje se 3D odgovor sistema u kapacitativnom domenu. U postupku generisanja 3D modela *pushover* površi polazi se od transformacije iz 2D ravanskog u 2D polarni koordinatni system, a zatim u 3D cilindrični koordinatni system, pa u 3D ortogonalni koordinatni sistem. Tipologija *pushover* krivih izvedena je u funkciji egzistencije linearnog, nelinearnog i kolapsnog subdomena, a takođe razmatranja su izvedena u funkciji nelinearne krutosti i klase duktilnosti sistema. Tipologija *pushover* površi izvedena je bazirajući se na generalizovanom modelu odgovora sistema preko parametara μ, μ_h i α, a na osnovu kojih je moguće formirati sisteme različite krutosti, nosivosti i duktilnosti. U slučaju varijacije parametara μ, μ_h i α kod *pushover* površi za jedan glavni pravac, razmatrani su slučajevi: identični parametri po smerovima jednog (svih) pravca i različiti parametri po smerovima jednog pravca. U slučaju varijacije parametara $\alpha>0$, $\alpha\approx0$, $\alpha<0$ i duktilnost $\mu=\mu_h$ i $\mu>\mu_h$ kod *pushover* površi za dva glavna pravca, razmatrani su slučajevi: identični parametri po smerovima jednog pravca i različiti parametri po smerovima jednog pravca. Komparacijom generisanih *pushover* površi identifikovane su četiri bitno različite grupe kod kojih je osnova definisana primenom četiri matematičke funkcije. Istraživanje prikazano u ovom radu predstavlja tipološke modele *pushover* površi na osnovu kojih je dalje moguće da se vrše razmatranja na realnim *pushover* površima 3D modela konstrukcija s kompleksnijom varijacijom odgovora sistema pri bidirekcionom dejstvu zemljotresa.

4 CONCLUSION

Analysis of the seismic response of structures using the pushover surface represents an original solution developed by the authors of this paper. By applying pushover surface it is possible to consider the response and performance of 3D models of structures in a more complex and complete way, especially in the case of emphasized non-symmetry of mass and stiffness, when exposed to bidirectional seismic action. The base setting from which the development of the mathematical formulation and generation of pushover surface started is the application of the pushover curve for the response of the system in one direction. By integrating the response of the system for a number of directions, or attack angles, the 3D system response in a capacity domain is presented. In the process of generating 3D model of the pushover surface one starts from the transformation of 2D plane to 2D polar coordinate system and then into the 3D cylindrical coordinate system and finally into the 3D orthogonal coordinate system. The typology of pushover curve is derived as a function of existence of the linear, nonlinear and collapse subdomains, and also considerations are made taking into account the nonlinear stiffness and ductility class of the system. The typology of pushover surface is derived according to the generalized model of the system's response through parameters μ, μ_h and α, based on which it is possible to create systems of different stiffness, strength and ductility. In the case of variations of parameters μ, μ_h and α of the pushover surface for one main direction, the following cases are considered: identical parameters along the senses of one (all) direction and different parameters along the senses of one direction. In the case of variation of parameters $\alpha>0$, $\alpha\approx0$, $\alpha<0$ and ductility $\mu=\mu_h$ and $\mu>\mu_h$ of the pushover surface for two main directions, the following cases have been discussed: identical parameters for both senses of one direction and different parameters for the senses of one direction. By comparing the generated pushover surfaces, four significantly different groups have been identified, in which the base was expressed by using four mathematical functions. The research presented in this paper defines typological models of pushover surfaces according to which the further analysis can be done on the real pushover surfaces of 3D models of structures with more complex variation of responses of the system during bidirectional earthquake actions.

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REZIME

TIPOLOGIJA NSPA PUSHOVER *KRIVIH I POVRŠI ZA 3D SEIZMIČKI ODGOVOR KONSTRUKCIJA PREMA PERFORMANSAMA*

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U radu je prikazana tipologija *pushover* krivih (*pushover curves*) i originalno razvijenih *pushover* površi (*pushover surfaces*), na osnovu generalizacije nelinearnog odgovora 3D sistema na seizmičku pobudu. Određivanjem i analizom NSPA (*Nonlinear Static Pushover Analysis*) *pushover* površi moguće je znatno kompleksnije i kompletnije razmotriti odgovor i performanse 3D modela konstrukcija izloženih bidirekcionom seizmičkom dejstvu. Osnovna postavka od koje se pošlo u razvoju matematičke formulacije i generisanju NSPA *pushover* površi jeste primena NSPA *pushover* krive za odgovor sistema u jednom pravcu. Integracijom odgovora sistema za veći broj pravaca, odnosno napadnih uglova pravaca dejstva zemljotresa, prikazuje se 3D odgovor sistema u kapacitativnom domenu. Tipologija NSPA *pushover* krivih izvedena je u funkciji egzistencije linearnog, nelinearnog i kolapsnog subdomena, a takođe su izvedena razmatranja u funkciji nelinearne krutosti i klase duktilnosti sistema. Tipologija NSPA *pushover* površi izvedena je bazirajući se na generalizovanom modelu odgovora sistema preko duktilnosti, duktilnosti u zoni ojačanja/omekšanja i koeficijenta odnosa nelinearne/linearne krutosti, a na osnovu čega je moguće formirati sisteme različite krutosti, nosivosti i duktilnosti. Istraživanje prikazano u ovom radu definiše tipološke modele NSPA *pushover* površi na osnovu kojih je dalje moguće vršiti razmatranja na realnim *pushover* površima 3D modela konstrukcija s kompleksnijom, a posebno nesimetričnom geometrijom, kao i varijacijom odgovora sistema pri bidirekcionom dejstvu zemljotresa.

Ključne reči: NSPA *pushover* krive i površi, tipologija, 3D performance, zemljotres

SUMMARY

TYPOLOGY OF NSPA PUSHOVER CURVES AND SURFACES FOR 3D PERFORMANCE-BASED SEISMIC RESPONSE OF STRUCTURES

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This paper presents a typology of pushover curves and the originally developed pushover surfaces based on the generalization of the nonlinear response of the 3D system to the earthquake action. By determination and analysis of the NSPA (Nonlinear Static Pushover Analysis) pushover surface, it is possible to obtain a more complex and complete insight of the response and performance of 3D models of structures exposed to the bidirectional seismic action. The setting which was a base for the development of mathematical formulation and generation of the NSPA pushover surface presents the application of NSPA pushover curve for the response of the system in one direction. By integrating the system responses for a number of directions, i.e. attack angles of directions of earthquake action, the presentation of 3D response of the system in the capacity domain is achieved. The typology of NSPA pushover curves is derived as a function of the existence of linear, nonlinear and collapse sub domain, and also considerations are made taking into account the nonlinear stiffness and ductility class of the system. The typology of NSPA pushover surface is derived based on the generalized model of the system response through ductility, ductility in hardening/softening zone and a coefficient of the relationship of stiffness in the nonlinear and linear domain, based on which it is possible to create systems of different stiffness, strength and ductility. The research presented in this paper defines the typological models of NSPA pushover surfaces which can be the base of further discussion on real pushover surfaces of 3D models of structures with a more complex, particularly non-symmetric geometry, as well as the variation of responses of the system due to bidirectional seismic actions.

Key words: NSPA pushover curves and surfaces, typology, 3D performance, earthquake