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PROBABILISTIC ANALYSIS OF BEARING CAPACITY OF PILES WITH VARIABLE PARAMETERS IN CPT TEST AND CALCULATION ACCORDING TO THE REQUIREMENTS OF EUROCODE 7 (EN 1997-1: 2004) REGULATIONS

PROBABILISTIČKA ANALIZA NOSIVOSTI STUBOVA SA PROMENLJIVIM PARAMETRIMA CPT ISPITIVANJA I PRORAČUNA U SKLADU SA PROPISIMA EUROKODA 7 (EN 1997-1:2004)

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Keywords
- pile
- bearing capacity
- CPT test
- probability theory
- Eurocode 7 (EN 1997-1:2004)

Abstract

A probabilistic concept for determining pile bearing capacity is presented, taking into account the variability of CPT test parameters and methodology of calculation according to the requirements of Eurocode 7 (EN 1997-1: 2004). Based on a single initial (real) CPT test, a larger number of generated (simulation) CPT tests are introduced drawn from solutions of statistics and probability theory. Research has found that the best solutions are achieved using the DA 2 design approach for n(CPT) ≥ 10 tests. Taking into account the deterministic and probabilistic approach in the analysis of pile bearing capacity, it is found that for the DA 2 design approach, the ratio of pile bearing capacity obtained from simulation and the capacity as determined through three methods (Mazurkiewicz, Van der Veen and hyperbolic approximation) is $R_{c,d}/P_u = 1.148$. *Using the reliability index, the following values of partial resistance factors are obtained:* $\gamma_{b,\beta} \approx 1.1$, $\gamma_{s,\beta} \approx 1.1$, *which also points to the DA 2 design approach.*

Ključne reči
- stub
- nosivost
- CPT ispitivanje
- teorija verovatnoće
- Eurokod 7 (EN 1997-1:2004)

Izvod

U ovom radu je prikazan probabilistički koncept određivanja nosivosti stubova, uzimajući u obzir varijabilnost parametara CPT ispitivanja i metodologiju proračuna u skladu sa propisima Eurokoda 7 (EN 1997-1:2004). Na osnovu početnog (stvarnog) CPT ispitivanja, uveden je veliki broj generisanih CPT ispitivanja (simulacija), koris–teći se rešenjima teorija statistike i verovatnoće. Istraži–vanje je otkrilo da se najbolja rešenja dobijaju primenom pristupa DA 2, za n(CPT) ≥ 10 ispitivanja. Uzimajući u obzir deterministički i probabilistički pristup analizi nosivosti stubova, utvrđeno je da u slučaju DA 2 pristupa odnos nosivosti dobijene simulacijom i nosivosti određene pomoću tri metode (Mazurkijevic, Van der Ven i hiperbolička aproksimacija) je jednak $R_{c,d}/P_u = 1.148$. *Primenom indeksa pouzdanosti, dobijene su sledeće vrednosti faktora delimične otpornosti:* $\gamma_{b,\beta} \approx 1.1$, $\gamma_{s,\beta} \approx 1.1$, *koje takođe upućuju na pristup DA 2.*

INTRODUCTION

Factors involved in determining and decision-making on the bearing capacity of piles are still burdened by a series of dilemmas, despite the sophisticated technology used when testing piles. The deterministic approach in identifying the bearing capacity of piles leaves the engineer-expert, as a final decision-maker, with a very limited space in the theory of decision making, given that one can only speak about the unique value of pile capacity. On the other hand, if the matter is discussed using the probability theory, then each individual value from the value spectrum has a specific probability of occurrence, with several of these values having the appropriate level of reliability. However, the key method in both of these approaches establishes a relation between the results of testing real pile models in real conditions (in-situ) with specific mathematical/engineering laws and standards for determining the pile bearing capacity. Thus, the generalization of the problem derived and proven on a number of tested piles with statistically acceptable error tolerances is an important factor as well.

Determining the bearing capacity of piles is performed by using analytical and numerical methods and tests on real pile models. The most reliable solutions for determining the bearing capacity of piles are obtained by using the *static load test* (SLT), whereby the testing program is carried out in real conditions and on a real pile model. The final result of the pile test, according to the SLT test, is a force-settlement curve, based on which the second stage determines the final value of the pile bearing capacity by using certain mathematical methods. The criteria and methods for determining pile bearing capacity, based on the solution obtained from SLT, have different mathematical formulations, which leads to different solutions. The most commonly used methods for determining the bearing capacity of piles based on SLT test are the following: Van der Veen's /1/, Mazurkiewicz's /2/, Decourt's /3/, Chin-Kondner's /4/ and the like. However, before initiation of the SLT test it is necessary to determine the value of the pile bearing capacity using analytical and/or numerical methods, which enables an adequate plan to be created in a timely manner for testing the bearing capacity of a real pile model. The procedure for implementing SLT is presented in detail in ASTM D 1143 /5/, which among other things provided a list of information that should be included in a test report. There is a well established term in engineering terminology – *'prediction of pile bearing capacity'*, which refers to the procedure for determining pile bearing capacity before the implementation of the SLT test. This includes: methods based on the use of laboratory and in-situ experiments with correlations, methods using simplified theoretical solutions and analytical procedures, and methods based on improved analytical and numerical procedures, /6/. Empirical methods (the first group) are based on principles of soil mechanics which include: methods of correlation from the *cone penetration test* (CPT); methods of correlation from the *standard penetration test* (SPT); and variations of dynamic methods pile capacity calculations. Methods that use simplified theoretical solutions and analytical procedures (the second group) consider the soil model as linear-elastic, brittle-plastic and nonlinear material. Numerical analyses (the third group) are in most cases based on the *finite element method* (FEM), but the *boundary element method* (BEM) and the *finite difference method* (FDM) are also applied.

This research is based on a probabilistic concept of bearing capacity of piles by introducing parameter variability in the CPT test. Solutions obtained using this approach are compared with those obtained from the SLT test.

EXISTING PILE CAPACITY SOLUTIONS ACCORDING TO SLT, CPT AND PROBABILISTIC CONCEPTS

Pile capacity testing using a number of different CPT-based approaches is shown in /7/, while paper /8/ presents a new CPT-based pile capacity test procedure. New expressions for the analysis of pile bearing capacity have been proposed in /9/, establishing correlation between values obtained from SLT and CPT tests. On the other hand, the study /10/ uses regression analysis which, based on the results from tests conducted on a significant number of piles, predicts the bearing capacity of the piles and soils of different properties. The paper /11/ compares the pile bearing capacity as determined using the CPT test and the pile base bearing capacity as determined using the SLT test. The development of modern methods for predicting the bearing capacity of piles is based on the fact that the bearing capacity of piles of different properties and different types of soil that contain the piles can be determined using mathematical formulations for a specific set of pile and soil types. A research which is based on this approach by applying support vector machine (SVM) and CPT test results is presented in /12/, while paper /13/ presents the research that links the least square support vector machine (LSSVM) and artificial intelligence. Given the certain degree of variation, and in many circumstances a pronounced level of unreliability (uncertainty), of a number of parameters involved in determining the bearing capacity of piles, these parameters also need to be included in the analysis of bearing capacity. In this sense one can speak about the reliability of obtained solutions for the bearing capacity of piles. Studies analysing the reliability of solutions based on the unreliability of input parameters are shown in /14, 15, 16/, taking into account the n-σ criterion /17/ and including the Bayesian theory in /16/ and /18/. Analyses of reliability of applying different methods in determining the bearing capacity of piles are shown in /19, 20/.

SLT TEST AND DETERMINING THE PILE BEARING CAPACITY; CPT TEST

Verification of methodology of pile capacity analysis presented in this study is based on the SLT test of a real pile model in actual conditions. Testing a pile using SLT test against vertical pressure force is conducted for the facility of SILOSI in the framework of TE Kostolac 'A' in Serbia, /21/. Figure 1 shows the situation with the position of the considered pile Š35 and research works. In terms of construction technology, it is a reinforced bored pile of D_p = 1 m diameter and L_p = 14 m length. For the purposes of research presented here, the key test is the cone penetration test labelled CPT-1 IMS, conducted in the immediate vicinity of the tested pile.

Figure 2 shows the force-settlement curve (test load curve) for the considered pile Š35, determined using the SLT test (system of concrete blocks and hydraulic press). During the SLT test, a constant pressure force is maintained throughout the entire load phase, with the following parameters being measured: the force that acts on the pile; pile head settlement; and the time soil consolidation, /22/. The experiment was first carried out for the load level of 2900 kN, then with the new load increment up to 3250 kN a higher increment of settlement is realized than in the first case. Ultimate values of capacity of the pile Š35 are also determined based on analytical procedure, by extrapolating the obtained force-settlement curves according to the following methods: Mazurkiewicz's (hyperbolic extrapolation) P_u = 3270 kN, Van der Veen's (exponential extrapolation) P_u = 3252 kN and hyperbolic approximation P_u = 3831 kN. The arithmetic mean of the results obtained based on the above methods (P_u = 3451 kN) is adopted as the ultimate capacity of pile Š35 tested using SLT.

In addition to testing the pile using SLT, the soil is also tested by using CPT. Figure 3a shows the CPT test diagram, where q_c is the soil resistance during the penetration of the cone, while q_s is the total friction along the skin. Soil classification is carried out using *Fuzzy* methodology in the CPeT-IT software, /23/ (Fig. 3b). The highest share (the highest likelihood of occurrence) is that of sandy soil (orange colour), while clay soil (clayey sand) is also present to some extent.

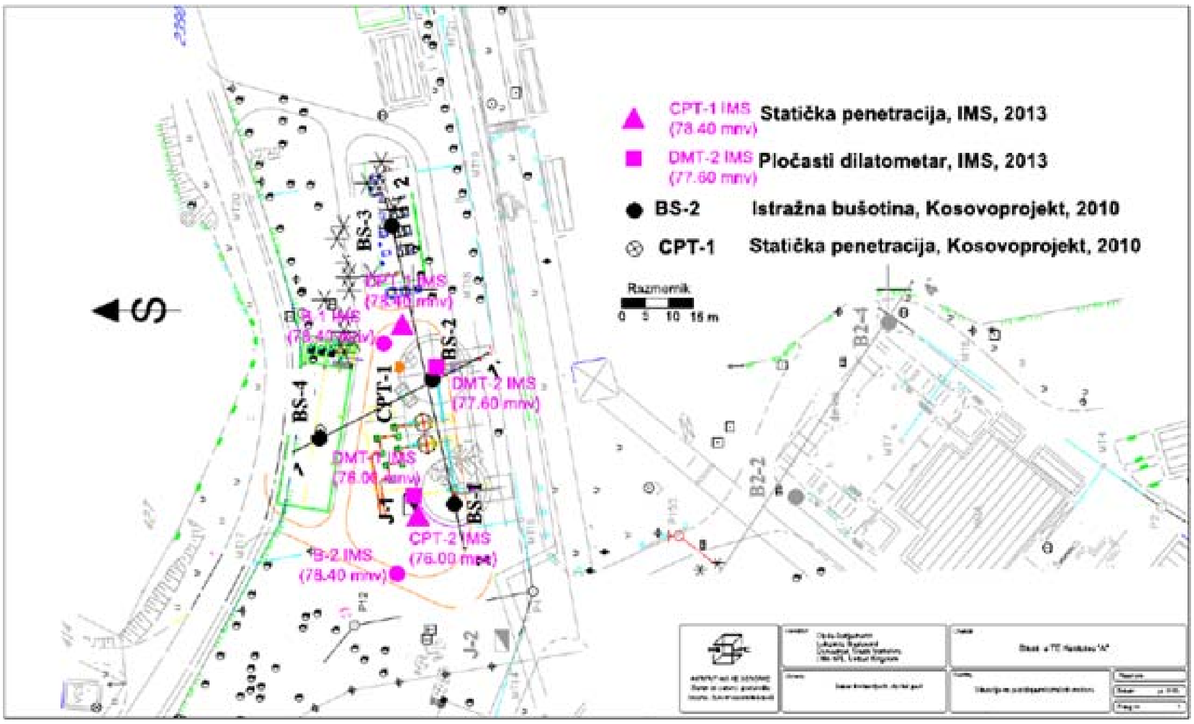


Figure 1. Situation with the position of the tested pile Š35 and research works, /7/.

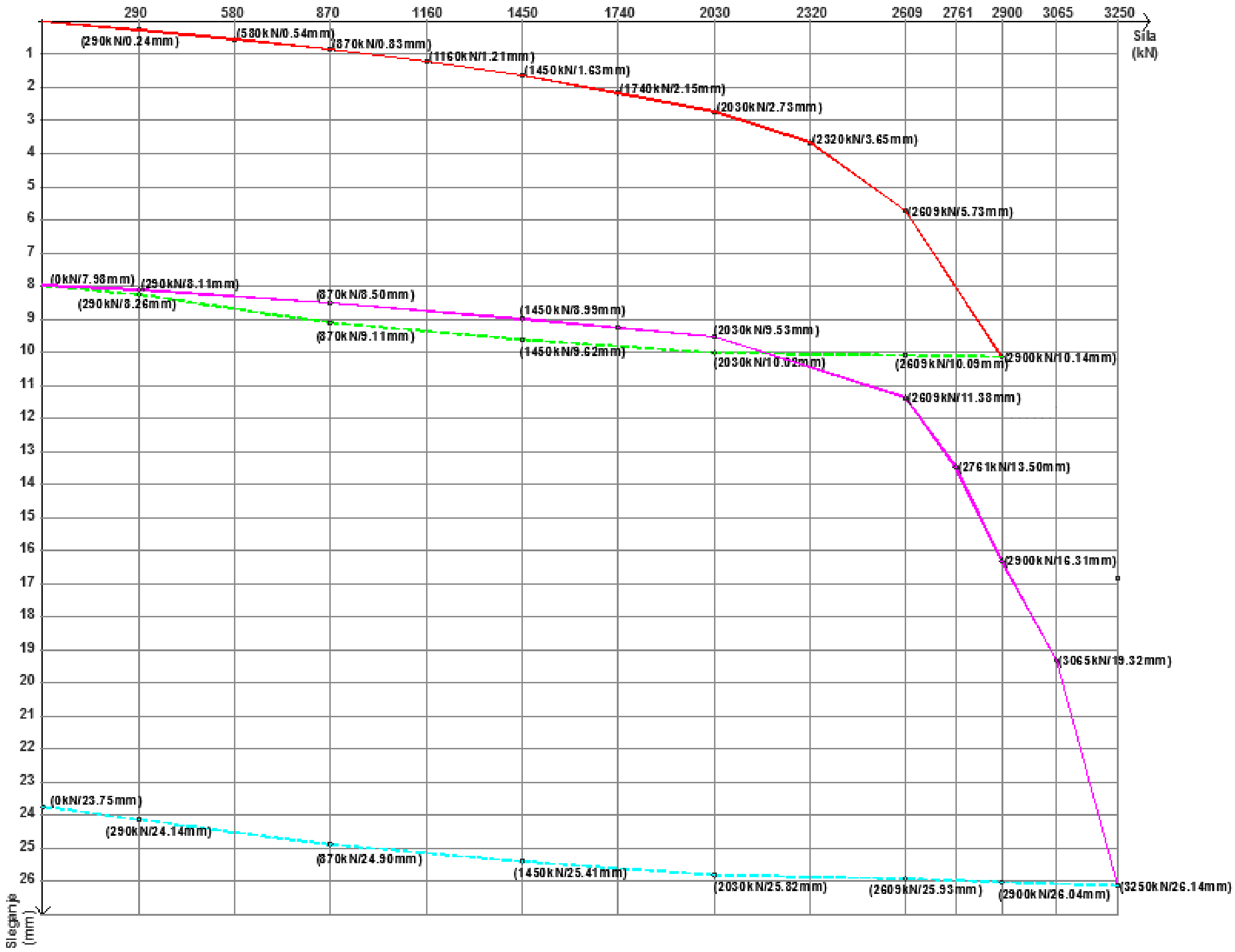


Figure 2. Force-settlement curve (test load curve) of the considered pile Š35, determined using the SLT test, /21/.

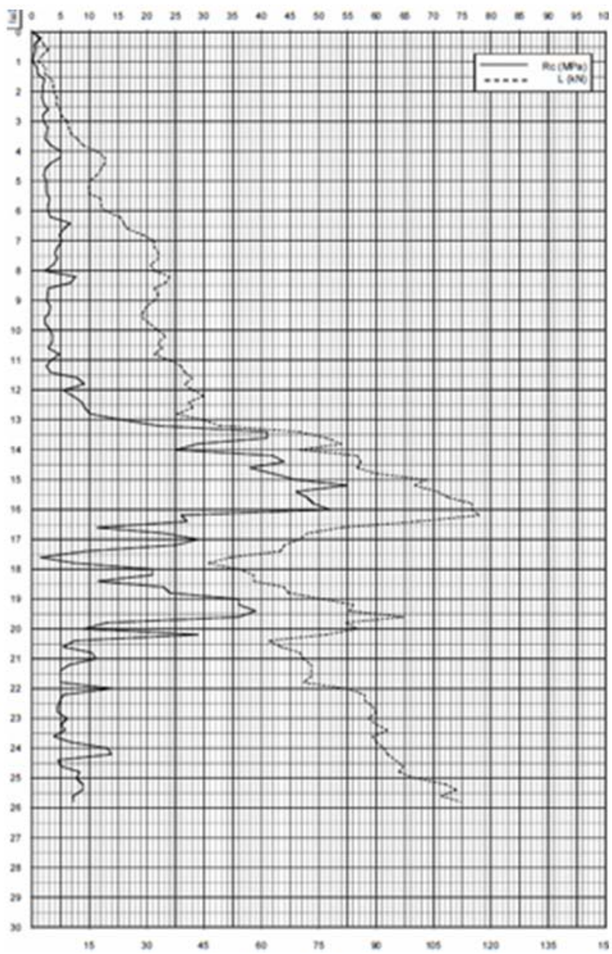
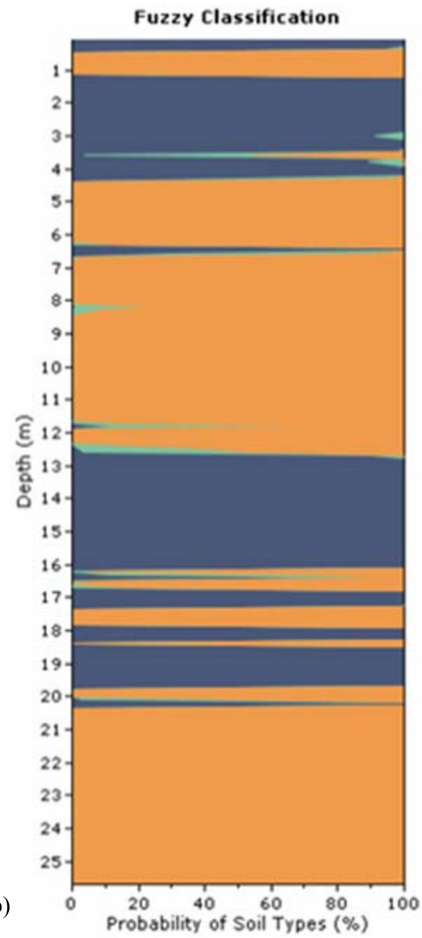
(a) 
(b) 

Figure 3. a) CPT test diagram /7/, b) soil classification using *Fuzzy* methodology in the CPeT-IT software /23/

PROBABILISTIC ANALYSIS OF PILE BEARING CAPACITY

The parametric pile capacity analysis presented in this part of the paper is based on Eurocode7 (EN 1997-1: 2004) /24/ requirements and probability theory. Starting from the initial CPT test (real CPT test), using statistical analyses and taking into account the variability of basic parameters of CPT test, a large number of CPT tests are generated (simulation CPT tests). Based on the results obtained from the initial (actual) CPT test, the total pile capacity R_c is determined as the aggregate value of capacity in the pile base R_b and pile skin R_s, /24/:

$$R_c = R_b + R_s \tag{1}$$

Given the larger number of generated (simulation) CPT tests introduced in addition to the initial (actual) CPT test, Eq. (1) becomes:

$$R_{c,i} = R_{b,i} + R_{s,i} \tag{2}$$

where $R_{c,i}$ is the total pile bearing capacity obtained based on the results of i-th simulation CPT test; $R_{b,i}$ is the capacity in pile base obtained based on the results of the i-th simulation CPT test, and $R_{s,i}$ is the capacity in pile skin obtained based on the results of the i-th simulation CPT test. Values of pile bearing capacity are reduced by a factor γ, by which capacities can be further reduced as a function of the CPT test (statistical evaluation):

$$R_{c,cal,i} = R_{b,cal,i} + R_{s,cal,i} = \frac{R_{b,i}}{\gamma} + \frac{R_{s,i}}{\gamma} \tag{3}$$

The capacity in the pile base obtained from results of the i-th simulation CPT test $R_{b,i}$ is determined from:

$$R_{b,i} = A_b p_b \tag{4}$$

where A_b is the area of pile base cross-section, while p_b:

$$p_b = 0.5\alpha_p \beta s \left(\frac{q_{c,I,mean} + q_{c,II,mean}}{2} + q_{c,III,mean} \right) \tag{5}$$

where α_p is the factor of pile class ($\alpha_p = 0.5$ bored pile); β is the factor of pile base expansion ($\beta = 1$); s is the factor of pile shape ($s = 1$); $q_{c,I,mean}$ is the mean value of $q_{c,I}$; $q_{c,II,mean}$ is the mean of the minimum value of cone resistance $q_{c,II}$; and $q_{c,III,mean}$ is the mean value of resistance $q_{c,III}$ according to Eurocode 7 (EN 1997-2: 2007), /25/:

$$\begin{aligned} q_{c,I,mean} &= \frac{1}{d_{crit}} \int_0^{d_{crit}} q_{c,I}\, dz \\ q_{c,II,mean} &= \frac{1}{d_{crit}} \int_{d_{crit}}^{0} q_{c,II}\, dz \\ q_{c,III,mean} &= \frac{1}{8D_{eq}} \int_0^{-8D_{eq}} q_{c,III}\, dz \end{aligned} \tag{6}$$

The capacity of pile skin $R_{s,i}$ is determined by:

$$R_{s,i} = O_p \int_0^{\Delta L} p_s dz \quad (7)$$

where O_p is the pile circumference, while:

$$p_s = \alpha_s q_{c,z,a} \quad (8)$$

where α_s is the friction factor along the pile skin; $q_{c,z,a}$ is the ultimate value of q_{cat} (z) depth. The minimum and the mean value of total capacity of the pile are determined based on its total capacity calculated from the initial (actual) CPT test and generated (simulation) CPT tests. The minimum value of total capacity of the pile $R_{c,k,min}$ obtained from all CPT tests is determined by:

$$R_{c,k,\min} = R_{b,k,\min} + R_{s,k,\min} \quad (9)$$

where:

$$R_{b,k,\min} = \min \frac{R_{b,cal,i}}{\xi_4}, \; R_{s,k,\min} = \min \frac{R_{s,cal,i}}{\xi_4} \quad (10)$$

The mean value of total capacity of the pile $R_{c,k,mean}$ obtained from all CPT tests is determined by:

$$R_{c,k,mean} = R_{b,k,mean} + R_{s,k,mean} \quad (11)$$

where:

$$R_{b,k,mean} = \frac{\frac{1}{N}\sum_{i=1}^{N} R_{b,cal,i}}{\xi_3}, \; R_{s,k,mean} = \frac{\frac{1}{N}\sum_{i=1}^{N} R_{s,cal,i}}{\xi_3} \quad (12)$$

Correlation factors ξ_3 and ξ_4 depend on the calculation method used (CPT test) and the number of generated (simulation) CPT tests. Figure 4 shows the change in correlation factors ξ_3 and ξ_4 as a function of the number of (simulation) CPT tests generated, n(CPT).

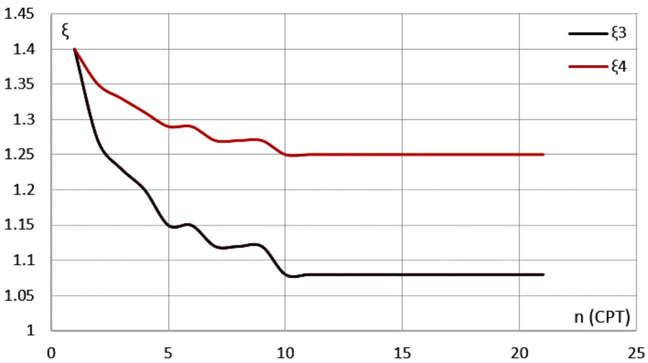


Figure 4. Change in correlation factors ξ_3 and ξ_4 as a function of the number of generated (simulation) CPT tests n(CPT).

It is evident that correlation factors for n(CPT) $\geq$ 10 become constant, while for a smaller number of CPT tests, values of these factors are considerably higher. The value of total bearing capacity of the pile from n(CPT) tests, R_c, is determined as the minimum value of $R_{c,k,min}$ and $R_{c,k,mean}$:

$$R_c = \min\left(R_{c,k,\min}, R_{c,k,mean}\right) \quad (13)$$

The value of total bearing capacity of the pile from n(CPT) tests, $R_{c,d}$, is reduced by the partial factors of pile base resistance γ band pile skin resistance γ_s as:

$$R_{c,d} = \min\left(R_{c,d,\min}, R_{c,d,mean}\right) \quad (14)$$

where:

$$R_{c,d,\min} = \frac{R_{b,k,\min}}{\gamma_b} + \frac{R_{s,k,\min}}{\gamma_s}$$
$$R_{c,d,mean} = \frac{R_{b,k,mean}}{\gamma_b} + \frac{R_{s,k,mean}}{\gamma_s} \quad (15)$$

According to Eurocode 7 (EN 1997-1: 2004) /24/ requirements, three design approaches (DA) should be taken into account when determining the pile bearing capacity: DA 1; DA 2; and DA 3. Each design approach introduces different partial resistance factors for pile base and skin. Since the research presented here is based on the analysis of capacity of bored piles, the partial resistance factors for design approaches are as follows: γ_b = 1.25; γ_s = 1 (DA 1); γ_b = 1.1; γ_s = 1.1 (DA 2); and γ_b = 1; γs = 1 (DA 3).

The introduction of variability of parameters in the pile capacity analysis, according to the concept of probability theory, is conducted for input parameters obtained from the CPT test. Variation of the soil resistance parameter is considered during the penetration of the cone $q_c(z)$ at a depth (z) through normal distribution (Gaussian distribution), whose probability density function (PDF), /26/, is:

$$f\left[q_c(z) \middle| q_{c,mean}(z), \sigma_c(z)\right] = \frac{1}{\sigma_c(z)\sqrt{2\pi}} e^{-\frac{\left[q_c(z)-q_{c,mean}(z)\right]^2}{2\sigma_c(z)^2}} \quad (16)$$

where $q_{c,mean}(z)$ is the mean value, and $\sigma_c(z)$ is standard deviation. First the analyses for $q_c(z)$ values are conducted based on normal distribution for the $[-n\sigma_c(z),+n\sigma_c(z)]$ interval and the $0.1[\sigma_c(z)]$ increment of standard deviation. The analysis is initiated by calculation of coefficient of variation C_v:

$$C_v = \frac{\sigma_c(z)}{q_{c,mean}(z)} \quad (17)$$

However, considering that the value of standard deviation $\sigma_c(z)$ during the initiation of calculation is unknown, the coefficient of variation C_v is determined as a percentage value k, so that standard deviation $\sigma_c(z)$ is determined as:

$$\sigma_c(z) = k q_{c,mean}(z) \quad (18)$$

The value of $\sigma_c(z)$ at (z) depth is defined so to be equal to $q_{c,mean}(z)$. In the next step, a set of $n\sigma_c(z)$ values in the interval of $[-3\sigma_c(z),+3\sigma_c(z)]$ is determined for each (z) value, and then an appropriate set of $[q_c(z) - 3\sigma_c(z), q_c(z) + 3\sigma_c(z)]$ values. For each of these values, PDF functions are determined in the following interval:

$$f\left[q_c(z) \middle| q_{c,mean}(z), -3\sigma_c(z)\right] \leq$$
$$\leq f\left[q_c(z) \middle| q_{c,mean}(z), \sigma_c(z)\right] \leq \quad (19)$$
$$\leq f\left[q_c(z) \middle| q_{c,mean}(z), +3\sigma_c(z)\right]$$

Interval $[q_c(z) - 3\sigma_c(z), q_c(z) + 3\sigma_c(z)]$ is chosen given that, according to the normal distribution, the function of random variable (RND) in this interval yields with the following probability:

$$P_r\left\{\left[q_{c,mean}(z) - 3\sigma_c(z)\right] \leq q_c(z) \leq \right.$$
$$\left. \leq \left[q_{c,mean}(z) + 3\sigma_c(z)\right]\right\} \approx 0.9973 \quad (20)$$

In the second step, using the function of random variable, simulation CPT tests are generated, obtaining thereby the simulated values of soil resistivity during the penetration of cone at (z) depth $q_{c,sim}(z)$:

$$\left[q_{c,mean}(z)-3\sigma_c(z)\right] \le \mathrm{RND}\left(q_{c,sim}(z)\right) \le \le \left[q_{c,mean}(z)+3\sigma_c(z)\right] \tag{21}$$

After that, based on Eq.(16), PDF functions of f[RND $(q_{c,sim}(z))|q_{c,mean}(z),\sigma_c(z)$] are also determined for these values. In total 20 simulation CPT tests are conducted.

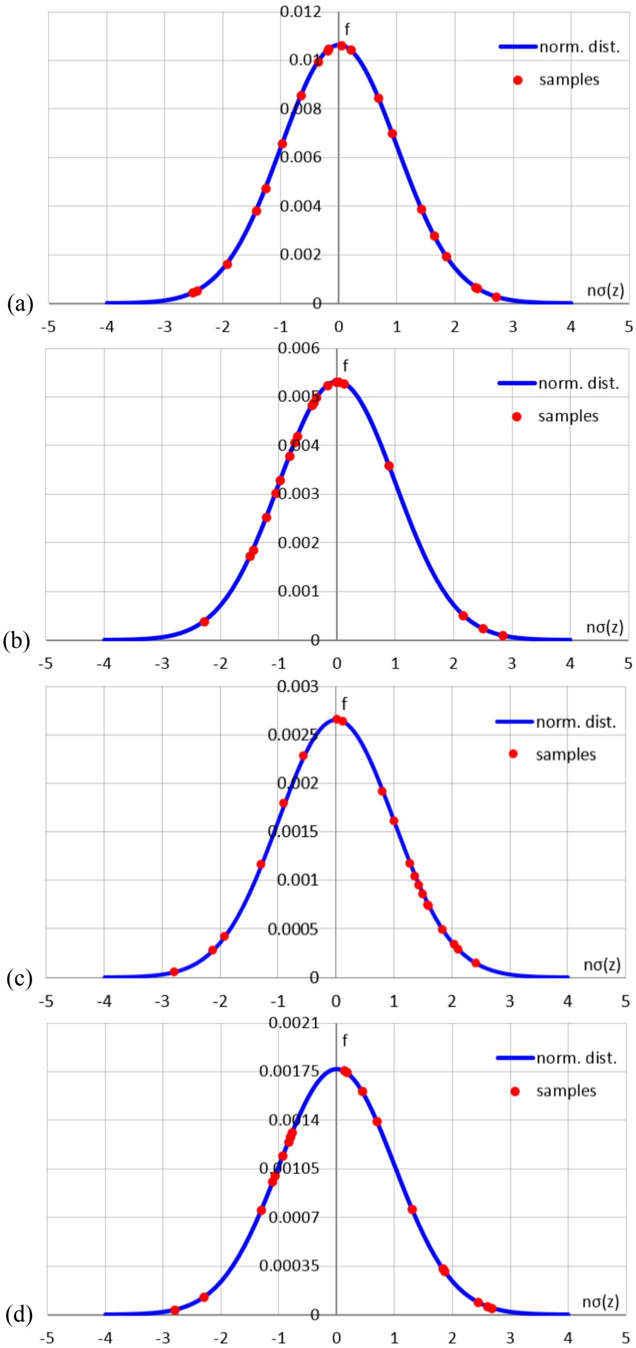


Figure 5. Probability density functions of standard normal distribution $f[q_c(z)|q_{c,mean}(z),\sigma_c(z)]$ and discrete values of generated simulation $f[\mathrm{RND}(q_{c,sim}(z))|q_{c,mean}(z),\sigma_c(z)]$ at a (z) depth (the abscissa shows normalized values of standard deviation $n\sigma_c(z)$): a) C_v = 5%, b) C_v = 10%, c) C_v = 20%, d) C_v = 30%.

NUMERICAL ANALYSES OF PILE BEARING CAPACITY

Based on the methodology formulated in the previous section, numerical analyses of bearing capacity of the pile are conducted. The coefficient of variation is considered as percentage value C_v = (5%, 10%, 20%, 30%), so that a total of 80 CPT simulation tests are generated. Figure 5 shows probability density functions of standard normal distribution of $f[q_c(z)|q_{c,mean}(z),\sigma_c(z)]$ and discrete values of generated $f[\mathrm{RND}(q_{c,sim}(z))|q_{c,mean}(z),\sigma_c(z)]$ at (z) depth, where the abscissa shows normalized values of standard deviation $n\sigma_c(z)$.

Figure 6 shows the diagrams of the original CPT test and generated CPT tests as a function of coefficient of variation C_v. It is evident that by increasing the values of coefficient of variation C_v a wider interval of soil resistance values can be covered during cone penetration $q_{c,sim}(z)$ along depth (z). Thus, the impact of variations in soil quality is taken into account for a given pile, or a given group of piles.

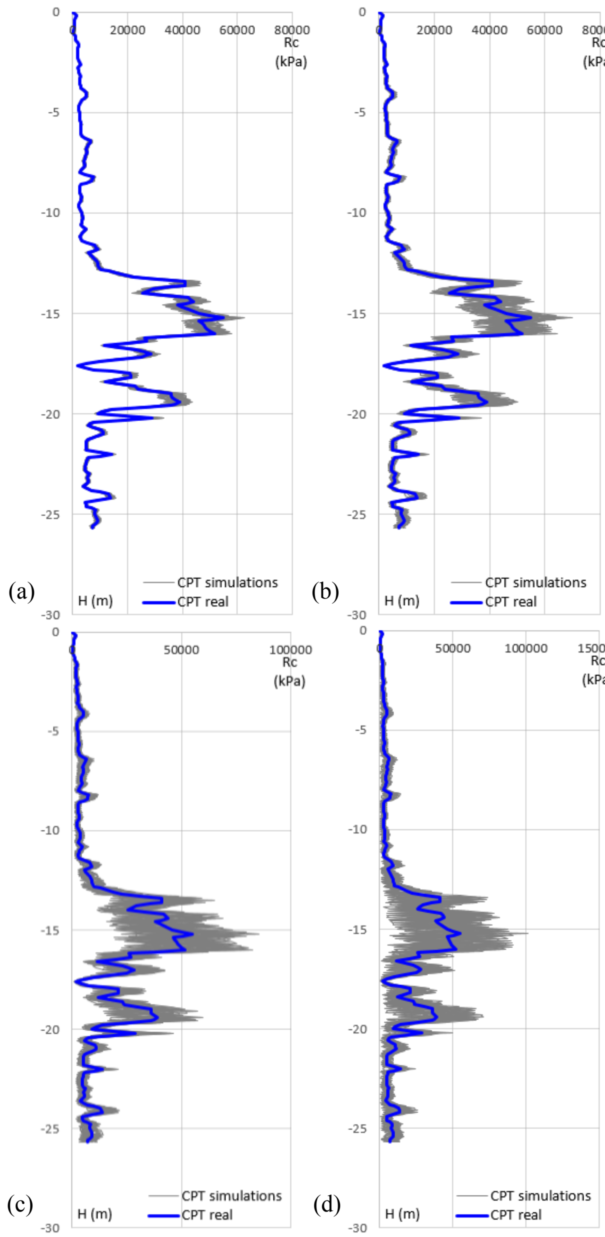


Figure 6. Diagrams of original and generated CPT tests: a) C_v = 5%, b) C_v = 10%, c) C_v = 20%, d) C_v = 30%.

The bearing capacity of the pile according to n(CPT) tests, $R_{c,d}$, is normalized by the ultimate bearing capacity of the SLT-tested pile P_u, shown in Fig. 7 as a function of design approaches DA 1, DA 2, and DA 3. In this way, changes in $R_{c,d} /P_u$ can easily be monitored, so that when $R_{c,d} /P_u < 1$ it can be said that the capacity of the pile as determined by CPT test simulations is lower than the ultimate capacity based on SLT test. The best solution ($R_{c,d} /P_u \approx 1$) is obtained for the second design approach DA 2 with partial resistance factors of $\gamma_b = 1.1$, $\gamma_s = 1.1$, and for $n(\text{CPT}) \geq 10$ tests. This clearly demonstrates the importance of conducting larger number of CPT tests or generating these based on the procedure described in the previous section.

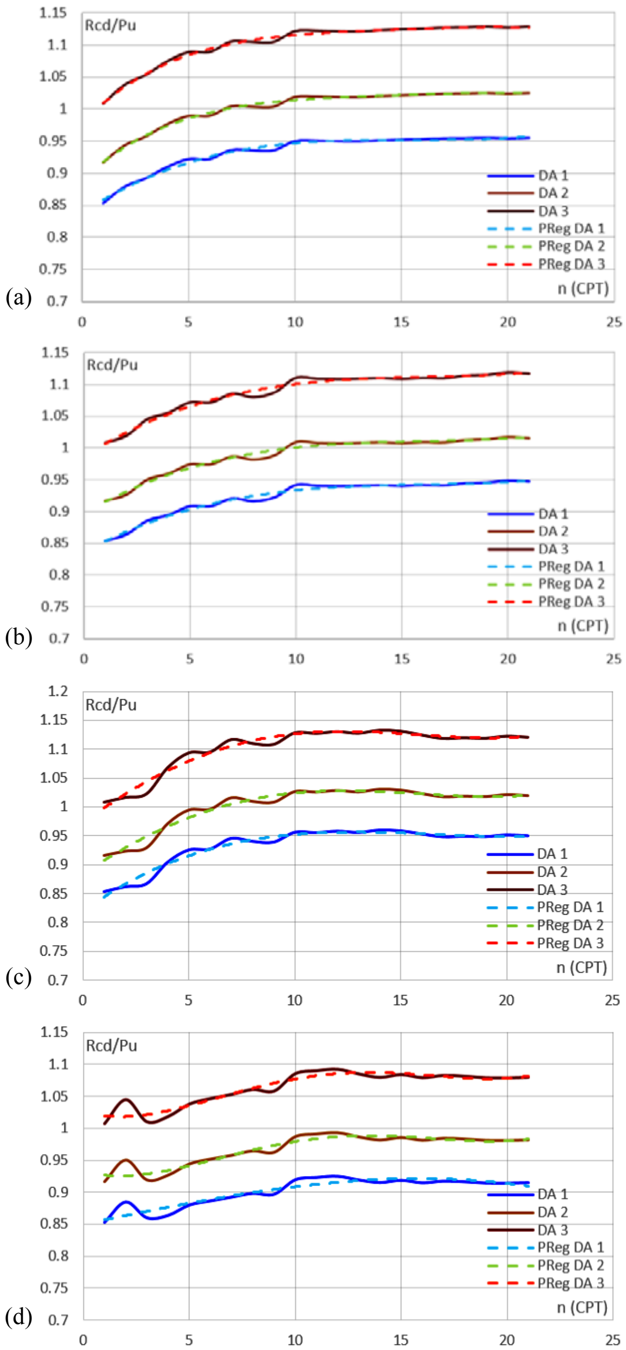


Figure 7. Normalized pile capacity values $R_{c,d}/P_u$ as a function of design approaches DA1, DA2 and DA3.

For each design approach regression analysis is conducted using a third order polynomial (PReg):

$$\text{PReg: } R_{c,d} / P_u = an(\text{CPT})^3 + bn(\text{CPT})^2 + cn(\text{CPT}) + d \quad (22)$$

where high values for the coefficient of correlation r^2 are obtained. Values of coefficients a, b, c and d are presented in Table 1.

Table 1. Coefficients a, b, c and d determined by regression analysis of a third-order polynomial (PReg).

C_v (%)	DA	a	b	c	d	r^2
5	1	0.00003	-0.0014	0.0221	0.8378	0.99
	2	0.00003	-0.0015	0.0237	0.8991	0.99
	3	0.00004	-0.0017	0.0260	0.9891	0.99
10	1	0.00002	-0.0011	0.0184	0.8359	0.98
	2	0.00002	-0.0012	0.0196	0.8969	0.98
	3	0.00003	-0.0013	0.0215	0.9866	0.98
20	1	0.00003	-0.0017	0.0270	0.8187	0.96
	2	0.00004	-0.0019	0.0297	0.8771	0.96
	3	0.00004	-0.0021	0.0326	0.9648	0.96
30	1	-0.000009	-0.000008	0.0068	0.8503	0.87
	2	-0.00005	0.000007	0.0073	0.9113	0.86
	3	-0.00001	0.000008	0.0080	1.0025	0.86

Analogous to the calculation of PDF function for CPT test parameters, PDF functions for the $R_{c,d} /P_u$ ratio are also calculated:

$$f\left[R_{c,d}/P_u \left| \left(R_{c,d}/P_u\right)_{mean}, \sigma_R\right.\right] = \frac{1}{\sigma_R\sqrt{2\pi}} e^{-\frac{\left[R_{c,d}/P_u - \left(R_{c,d}/P_u\right)_{mean}\right]^2}{2\sigma_R^2}} \quad (23)$$

where $(R_{c,d} /P_u)_{mean}$ is the mean value, and σ_R is standard deviation. Figure 8 shows the probability density functions of normal distribution $f[R_{c,d} /P_u|(R_{c,d} /P_u)_{mean}, \sigma_R]$ and the discrete values of normalized pile capacity $f[R_{c,d,sim}/P_u|(R_{c,d}/P_u)_{mean}, \sigma_R]$ as a function of design approaches DA 1, DA 2 and DA 3. The deviation of calculated discrete values from the simulation is very small and it is almost located at the probability density function of normal distribution.

In order to determine the probability so that the calculated $R_{c,d} /P_u$ ratio is lower than 1, the function of cumulative probability of normal distribution (CDF - cumulative distribution function) is determined according to /16/, with an additional adjustment for the purpose of this research:

$$F\left[R_{c,d}/P_u \left| \left(R_{c,d}/P_u\right)_{mean}, \sigma_R\right.\right] = P\left(\left(R_{c,d}/P_u\right) \leq 1\right) = \int_{-\infty}^{R_{c,d}/P_u=1} \frac{1}{\sigma_R\sqrt{2\pi}} e^{-\frac{\left[R_{c,d}/P_u - \left(R_{c,d}/P_u\right)_{mean}\right]^2}{2\sigma_R^2}} dR \quad (24)$$

Figure 9 shows the cumulative probability function of normal distribution $F[R_{c,d} /P_u|(R_{c,d} /P_u)_{mean},\sigma_R]$ and discrete values of normalized pile capacity $F[R_{c,d,sim}/P_u|(R_{c,d}/P_u)_{mean},\sigma_R]$ as a function of design approaches DA 1, DA 2 and DA 3. The calculated values of probability of P for $R_{c,d}/P_u = 1$, and the identified values of $R_{c,d} /P_u$ for the probability of $P = 1$ across all design approaches (DA 1, DA 2, DA 3) and coefficients of variation C_v are shown in Table 2.

When the pile capacity test is conducted for a known value of probability of $P = 1$, then the best solution is obtained for design approach DA 1 across all coefficients of variation C_v, whereby the mean value for $R_{c,d}/P_u$ is 1.085. When the pile capacity test is conducted for a known value of $R_{c,d}/P_u = 1$, then the best solution is also obtained for design approach DA 1 across all coefficients of variation C_v, whereby the mean value for P is 0.99. However, this analysis shows that the necessary condition is fulfilled, but it is insufficient.

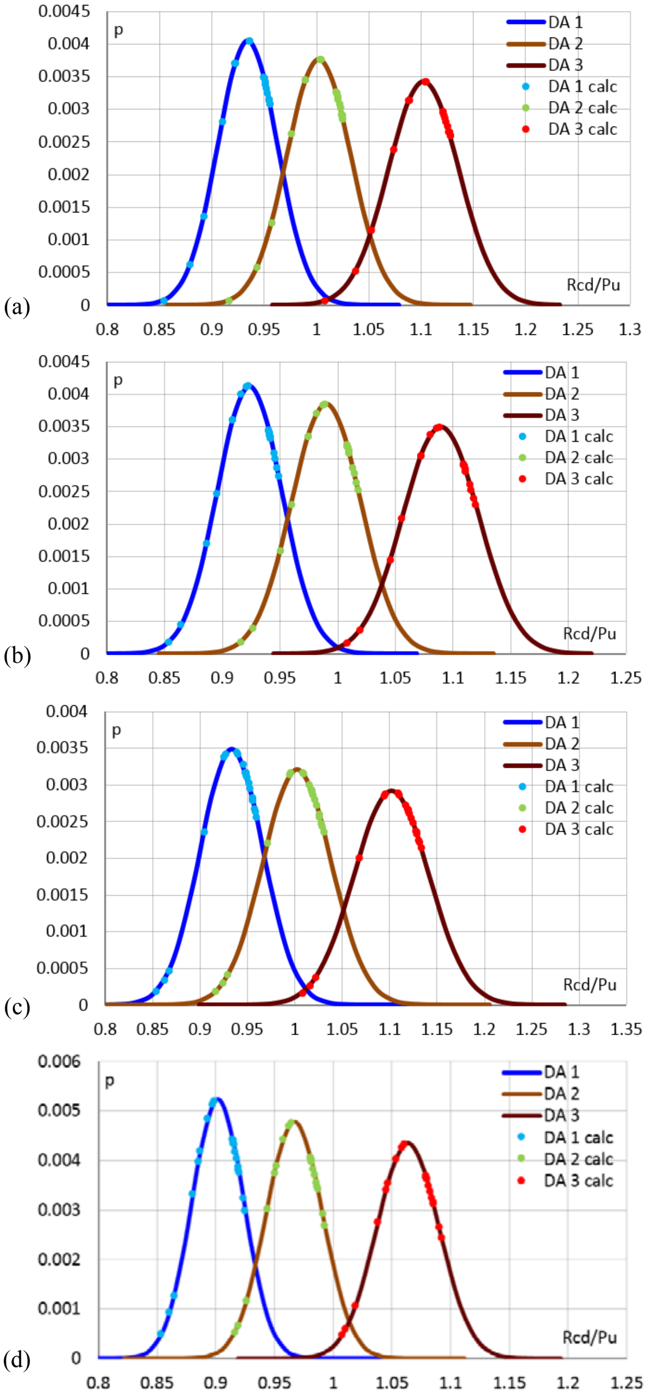


Figure 8. Probability density functions of normal distribution $f[R_{c,d}/P_u|(R_{c,d}/P_u)_{mean},\sigma_R]$ and discrete values of normalized pile capacity $f[R_{c,d,sim}/P_u|(R_{c,d}/P_u)_{mean},\sigma_R]$: a) C_v=5%, b) C_v=10%, c) C_v=20%, d) C_v=30%

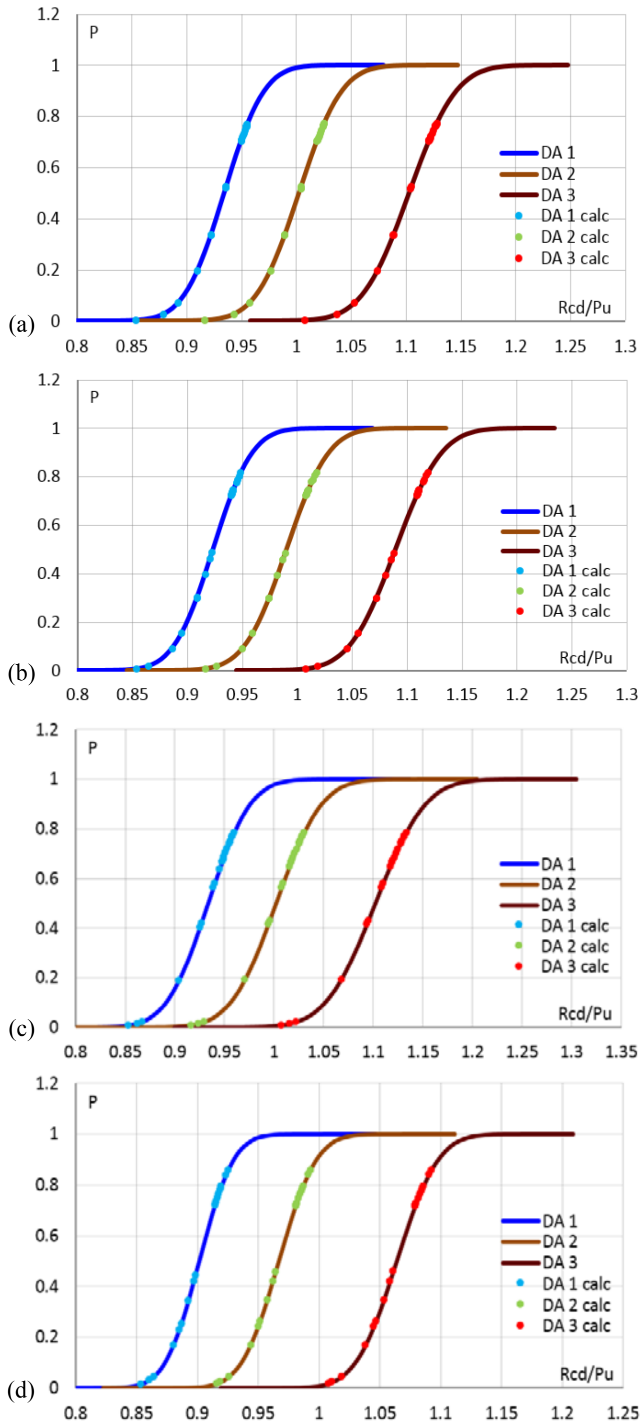


Figure 9. Cumulative probability function of normal distribution $F[R_{c,d}/P_u|(R_{c,d}/P_u)_{\mathrm{mean}},\sigma_R]$ and discrete values of normalized pile bearing capacity $F[R_{c,d,sim}/P_u|(R_{c,d}/P_u)_{mean},\sigma_R]$: a) $C_v = 5\%$, b) $C_v = 10\%$, c) $C_v = 20\%$, d) $C_v = 30\%$.

By combining design approach DA 2 as the optimum solution obtained based on the deterministic approach with the solution obtained based on the probability theory, the mean $R_{c,d}/P_u$ value for the known probability value of $P = 1$ is 1.147, while the mean value of probability P for the known $R_{c,d}/P_u$=1 is 0.62. This indicates a probability of $P = 0.62$ in the case when the ultimate capacity of the tested pile Š35 is determined by using the arithmetic mean of the results of three methods (*Mazurkiewicz*, *Van der Veen* and hyperbolic approximation) as P_u = 3451 kN. The probability is P = 1 when the ultimate bearing capacity of tested pile Š35 is $R_{c,d}$ = 1.147, P_u = 3960 kN according to this procedure.

Table 2. Calculated values of probability P for $R_{c,d}/P_u$ = 1 and determined values of $R_{c,d}/P_u$ = 1 for probability of P = 1 across all design approaches (DA 1, DA 2 and DA 3) and variations of C_v.

		P (known)		$R_{c,d}/P_u$ (known)	
C_v (%)	DA	P	$R_{c,d}/P_u$	P	$R_{c,d}/P_u$
5	1	1	1.08	0.99	1
	2	1	1.15	0.47	1
	3	1	1.25	0.001	1
10	1	1	1.07	1	1
	2	1	1.13	0.63	1
	3	1	1.23	0.004	1
20	1	1	1.14	0.97	1
	2	1	1.2	0.48	1
	3	1	1.31	0.005	1
30	1	1	1.05	1	1
	2	1	1.11	0.91	1
	3	1	1.21	0.01	1

Considering Eurocode 7 (EN 1997-1: 2004) /8/ recommendation that the ultimate pile bearing capacity, using the SLT test, should be determined for the level of settlement equal to 10% of pile diameter, the appropriate settlement is $\Delta = 10\%D_p$ = 10 cm. The SLT test yielded with a slightly lower value of maximum soil settlement and pile ultimate bearing capacity, so that the capacity of pile under increased load can be expected to increase to some extent (≈ 14.7%), which would correspond to the calculated value of $R_{c,d}$ = 3960 kN. Also in the case of solution for design approach DA 1, the use of probability theory yielded with 8.5% increase in ultimate bearing capacity ($R_{c,d}$ = 3744 kN).

After analysing the capacity of the pile, a reliability analysis is conducted in order to determine (check) the partial resistance factors of pile base γ band pile skin γ_s. The reliability index β is defined as a function of the mean pile capacity value, the arithmetic mean of the results obtained by three methods (Mazurkiewicz, Van der Veen and hyperbolic approximation) and standard deviation:

$$\beta = \frac{R_{c,d,mean} - P_u}{\sigma_R} \tag{25}$$

Table 3. Calculated values of reliability index β across all design approaches (DA 1, DA 2 and DA 3) and coefficients of variation C_v and the corresponding partial resistance factors $\gamma_{b,\beta}$ and $\gamma_{s,\beta}$

C_v (%)	DA	β	$\gamma_{b,\beta}$	$\gamma_{s,\beta}$
5	1	-2.31	1.105	1.097
	2	0.08		
	3	3.04		
10	1	-2.75	1.089	1.089
	2	-0.33		
	3	2.7		
20	1	-2.01	1.104	1.098
	2	0.06		
	3	2.59		
30	1	-4.46	1.063	1.063
	2	-1.36		
	3	2.4		

Table 3 shows calculated values of reliability index β across all design approaches (DA 1, DA 2 and DA 3) and coefficients of variation C_v. Positive and higher reliability index values are of interest for consideration.

However, to determine the value of partial resistance factors of the pile base γ band pile skin γ_s it is necessary to consider the zero value of reliability index β and corresponding design approaches DA. Minimum required partial resistance factor for $\beta = 0$ is determined by interpolating reliability index β and corresponding partial resistance factors γ band γ_s. The mean values of minimum required partial resistance factors for $\beta = 0$ are $\gamma_{b,\beta}$ = 1.09 for pile base and $\gamma_{s,\beta}$ = 1.087 for pile skin. It is evident that these values are close to 1.1.

CONCLUSION

The amounts of mutual deviations of pile bearing capacities obtained using different methods of calculation are several percent to tens of percent. The key question is to what level of load the testing should be conducted in order to reliably determine the value of pile capacity. Given that the SLT test requires considerable time for preparation of the test itself and a space that should be provided in order for the test to be conducted well and safely, this paper presents a procedure which can determine the pile capacity relatively quickly based on probabilistic analysis and applying a single CPT test.

The study found that the best solutions are achieved by applying the design approach DA 2 with partial resistance factors γ_b = 1.1, γ_s = 1.1, and for the number of simulation CPT tests n(CPT) ≥ 10. By using a reliability index, the paper also pointed to the advantage of design approach DA 2. A set of pile capacity values is determined using the probability density function of normal distribution PDF and cumulative probability function of normal distribution CDF. When combining deterministic and probabilistic approaches in analysing the pile bearing capacity, the $R_{c,d}/P_u$ = 1.147 ratio is found to be authoritative for design approach DA 2. Based on the developed CDF curves it is possible to establish relations for other levels of probability P (higher or lower) as well, and it is also possible to consider higher and lower levels of reliability index. Values of partial resistance factors $\gamma_{b,\beta} \approx 1.1$, $\gamma_{s,\beta} \approx 1.1$ are determined by analysing the reliability index, which also points to the authoritative design approach DA 2.

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ESIS ACTIVITIES

CALENDAR OF CONFERENCES, TC MEETINGS, and WORKSHOPS

May 24-27, 2016 May 23-24, 2016	NT2F16 and Summer School	Dubrovnik, Croatia	http://nt2f16.fsb.hr
June 1-3, 2016	11th Int. Conf. on Multiaxial Fatigue & Fracture (ICMFF11)	Seville, Spain	http://www.icmff11.es/
June 20-24, 2016 June 18-19, 2016 June 22, 2016	21st Europ. Conf. on Fracture (ECF21) Summer School ESIS TC-13 Meeting	Catania, Italy	http://www.ecf21.eu/
September 5-7, 2016	XVIII International Colloquium Mechanical Fatigue of Metals	Gijón, Spain	http://icmfm18.org/
November 8-9, 2016	ESIS TC-24 Meeting	Leoben, Austria	http://www.mcl.at/
June 18-23, 2017	ICF14-Fourteenth International Conference on Fracture	Rhodes, Greece	http://www.icf14.org/
September 4-7, 2017	2nd Int. Conf. on Structural Integrity (ICSI2)	Madeira, Portugal	http://icsi.inegi.up.pt/
September 10-14, 2017	ESIS TC-4 Meeting	Les Diablerets, Switzerland	
September 19-22, 2017	3rd Int. Symp. on Fatigue Design and Material Defects	Lecco, Italy	http://www.fdmd3.polimi.it
September 25-27, 2017	ESIS TC-5 Meeting	St. Petersburg, Russia	
August 26-31, 2018	22nd European Conference of Fracture (ECF22)	Belgrade, Serbia	