DAMPING MODELS FOR FLOW CHART BASED STRUCTURAL ANALYSIS

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***Abstract**: This paper presents the aspects of damping modelling in structural analysis through the systematization of damping types and flowcharts, depending on the type of analysis applied: linear and non-linear, static and dynamic. Damping has been systematized based on the way it was introduced into calculations, i.e. over material damping, link element damping and damping directly introduced into the analyses which are conducted in capacitive, time and frequency domains. In the process of creating numerical structural models, the type of damping and the way of its introduction into structural analysis can be very efficiently selected by applying the flow charts developed. By applying the developed flowcharts, alternative approaches to the introduction of damping into structural analysis can also be defined.*

***Keywords**: Damping, structural analysis, flowchart.*

1. Introduction

Calculating a building's structure requires analyzing the key parameters, modelling the structure and selecting the method of analysis based on which the dynamic properties and forces in the elements' cross-section and strain are to be determined. The purpose of numerical modelling of the structure is to define the elements' geometric and material properties and the systems' load and damping. These properties and loads make the basis for forming the bars' stiffness and mass matrices, and subsequently the stiffness and mass matrices of the entire system. The system's mass is to be calculated by converting loads to masses or by applying masses directly to the system. The choice of system's damping parameter depends on the type of materials of the element/structure. The quantitative value of the coefficient of relative damping and other coefficients are determined in the standard procedure. However, in addition to be directly defined, damping may be also introduced into the structural analysis indirectly.

Structural dampings are determined based on experimental, analytical and numerical studies. In many experiments damping is determined from free damped oscillations as a function of logarithmic decrement. This damping is in the domain of linear-elastic behaviour. If the system's displacement is induced by a strong excitation that makes it

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oscillate (effects of earthquake or strong wind), hysteretic damping should also be taken into considered, which is the consequence of the development of non-linear strain. Experimental results have enabled the introduction of analytical expressions based on the use of specific matrix multipliers, so that damping is introduced either explicitly or implicitly. Numerical studies also use analytical solutions so that damping is introduced into the system through a series of coefficients. Software products which also incorporate non-linear system behaviour enable hysteretic damping to be determined based on energy dissipation through cycles of non-linear behaviour (hysteretic loops). Some software solutions enable free damped vibrations to be analyzed by simulating the system's out-of-balance state and monitoring its response in the time domain. Damping has been most commonly introduced into structural analysis as part of the critical damping whose values depend on the type of material, and not on the mass and stiffness of the system [1]. By applying the equivalent relative damping coefficient, damping can be analyzed in different types of materials, being introduced in the form of composite damping [2]. Using a unitary equivalent relative damping coefficient in non-linear structural analysis, both viscous and hysteretic damping can be taken into account [3]. In addition to quality of materials, the value of relative damping coefficient is affected by vibration amplitudes and periods, eigenforms of vibration, types of links and the structure's configuration [4]. Paper [5] considers different types of damping to be introduced into structural analysis: material/inertial, ultimate/structural and fluid/viscous damping. Construction systems are considered as continuous and discrete.

This paper provides a systematization of damping types in structures by creating flowcharts and modelling depending on the type of analysis applied: linear and non-linear, static and dynamic. The aim of this study is to systematize the aspects and define the algorithms of damping modelling in analyses of structures in capacitive, time and frequency domains. The following analyses were considered: *spectral-modal analysis* (SMA), *non-linear static pushover analysis* (NSPA), *linear dynamic analysis* (LDA), *non-linear dynamic analysis* (NDA), *steady-state analysis* (SSA) and *power spectral density analysis* (PSDA). The paper also seeks to answer the question: whether and when the damping of link elements adds up with overall damping? The paper also presents a way of defining damping depending on the type of link elements.

2. General approach to damping in structural analysis

In the process of modelling the structure and preparing the analysis based on which the structure will be calculated, damping can be introduced via material damping, link element damping and damping which is directly defined in the analysis. Figure 1 shows the flowchart of the general approach to damping in structural analysis.

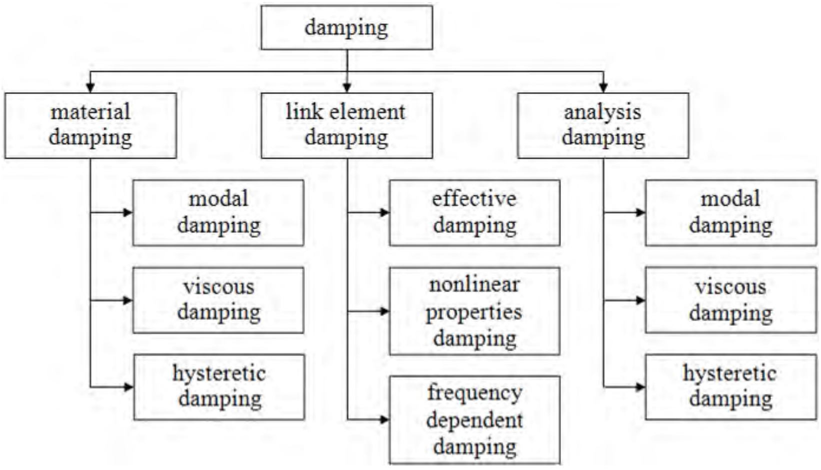

Fig. 1. Flowchart of the general approach to damping in structural analysis

Material damping is introduced with the definition of the type of material and can be applied to a particular group of line, surface or spatial finite elements. This principle of introducing damping into the analysis is very advantageous, given that enables damping to be also defined for structures consisting of segments made of different types of materials. It also enables infill and composite structures to be modelled. It particularly enables defining damping for the supporting structure, specifically for soil (radiation damping, whereby soil is modelled by using spatial solid finite elements).

Depending on the type of analysis for which material damping is being defined, damping can be generally divided into modal, viscous, and hysteretic damping. Modal damping is introduced into *spectral-modal analysis* and *linear/non-linear modal dynamic analysis*. Viscous damping is introduced into *linear/non-linear dynamic analysis* which requires numerical integration to be carried out, while hysteretic damping is introduced into *steady-state analysis* and *power spectral density analysis*.

Depending on the type of the link element, damping can be introduced as: effective damping, damping in non-linear behaviour and damping of frequency dependent link elements. Effective damping is introduced into *spectral-modal analysis*, *linear dynamic analyses* (modal and numerical integration), *steady-state analysis* and *power spectral density analysis*. In frequency dependent link elements damping is introduced into *steady-state analysis* and *power spectral density analysis*.

Depending on the type of analysis, damping can be generally divided to: modal, viscose and hysteretic damping. Each of the damping can be introduced by applying different procedures presented later in this paper.

2.1. Material damping

Material damping, in the form of modal damping, is introduced using the relative damping coefficient ξ_m for different types of materials, which represents the ratio of actual and critical damping. This damping is also known as composite modal damping, and its values are within the limits of $0 \leq \xi_m \leq 1$. Material damping, in the form of a viscous (proportional) damping, is introduced by applying the factors of participation of the system's mass and stiffness, so that the damping matrix is calculated as follows [6]:

(1) $$[C] = \alpha[M] + \beta[K],$$

(2) $$\alpha = 4\pi \frac{T_1\xi_1 - T_2\xi_2}{T_1^2 - T_2^2}, \qquad \beta = \frac{1}{\pi} T_1 T_2 \frac{T_1\xi_2 - T_2\xi_1}{T_1^2 - T_2^2},$$

where α and β are factors of participation of mass and stiffness matrices in the system's damping matrix, T_1 and T_2 are periods of vibration for the first and second eigenform, ξ_1 and ξ_2 are relative damping coefficients for the first and second eigenform. Material damping, in the form of hysteretic damping, is introduced by applying the factors of participation of the system's mass and stiffness, analogous to the principle of introduction of viscous damping. Given that this damping is being introduced in analyses in the frequency domain, the calculation uses the hysteretic damping matrix [6]:

(3) $$[D] = \omega[C].$$

2.2. Damping induced by link elements

Damping induced by link elements, which are modelled in linear analyses, is defined through effective damping c_{eff}. This effective damping is introduced individually for each link element and independently for each of the 6 damping components. It can also be used for representing, among the others, energy dissipation due to non-linear damping and development of plastic strains. Effective damping is determined analogous to determining the components of the effective stiffness. If the link element is defined based on the possibility of developing non-linear strains, then dissipation of hysteretic energy in link elements is required to be calculated during the non-linear analysis. On the other hand, in the case of non-linear behaviour of link elements, damping can be additionally introduced depending on the type of the link element. In the case of damper element, the relation between non-linear forces and displacements is as follows [6]:

(4) $$f_d = cv^e,$$

where f_d is the force in the damper element, c is the damping coefficient ($c=\xi c_c$ - product of the relative and the critical damping coefficient), v is the strain rate in the damper element, e is the damping exponent ($0.2\le e\le 2$). In friction-pendulum isolator, damping is introduced in the axial force analysis f_i:

(5) $$f_i = kd + cv,$$

where k is the isolator stiffness, d is the isolator displacement, while the relative damping coefficient ξ can be determined from:

(6) $$\xi = \frac{c}{2\sqrt{km}},$$

where m is the corresponding isolator mass. In the case of double-acting friction-pendulum isolator, this damping is introduced in axial force analysis through:

(7) $$f_i = cv + \begin{cases} k_c\left(d+\Delta_c\right) & \left(d+\Delta_c\right)<0 \\ k_t\left(d-\Delta_t\right) \quad \text{if} & \left(d-\Delta_c\right)>0 \\ 0 & \text{other} \end{cases},$$

where k_c is the isolator's compressive stiffness, k_t is the isolator's tensile stiffness, Δ_c is the clearance gap under compression and Δ_t is the clearance gap under tension. In the case of other types of link elements which are based on hysteretic behaviour, such as multilinear plastic, *Wen* and rubber isolators, which can be applied in non-linear analyses, damping is explicitly not introduced into calculation, but is determined during the analysis.

Damping of frequency dependent link elements is used in analysis in the frequency domain, where the complex impedance is represented by the frequency dependent properties. The real part corresponds to stiffness, while the imaginary part corresponds to hysteretic damping. Frequency dependent properties of the link element with six degrees of freedom can be expressed in matrix form (36 elements), wherein the element of the impedance matrix [7]:

(8) $$z_i = k_i + ic_i,$$

where k_i is the stiffness component for the i-th degree of freedom, c_i is the damping component for the i-th degree of freedom.

2.3 Damping that is directly defined in the analysis

Modal damping that is directly defined in the analysis can be introduced as: constant damping for all modes, interpolated damping by period or frequency and applying the factors of mass and stiffness proportional damping by coefficient. Constant damping for all modes is defined by applying a unique relative damping coefficient of ξ_c. If only one type of material is defined in the process of structural modelling, then damping that is introduced through the material ξ_m becomes equivalent to damping which is introduced as a constant damping in the analysis of ξ_c. However, it is necessary to take into account that these differently introduced types of damping are added up, so that overall damping will be further increased. Interpolated damping ξ_i is defined as a function of the selected vibration T_i or frequency f_i periods. Here, for certain periods of vibration (frequency) it is possible to separately define the relative damping coefficients, and then, using interpolation, to determine the corresponding relative damping coefficients $\xi_{i,i}$ for the calculated periods of vibrations (frequencies).

Outside the defined region, where damping is predefined, the value of the relative damping coefficient is constant. Some software solutions allow defining factors α and β directly or defining periods of vibration of the first and second eigenform T_1 and T_2, and the corresponding values of relative damping coefficients ξ_1 and ξ_2, which is then followed by the calculation of factors α and β. Damping matrix can be calculated using the following formula [6]:

(9) $$[C] = \alpha[I] + \beta[\Omega^2],$$

where $[\Omega^2]$ is the matrix of squares of the system's eigenvalues and $[I]$ is the unit matrix.

Viscous damping, which is directly defined in the analysis, can be introduced by: using factors of mass and stiffness participation (α and β) as a function of vibration periods of the first and second eigenform T_1 and T_2 (specify damping by a period) and as a function of frequencies of the first and second eigenform f_1 and f_2 (specify dumping by frequency). Introducing damping by using different relative damping coefficients for the first two eigenforms (frequency) has a number of advantages over the principle of using a unique relative damping coefficient.

Hysteretic damping, which is directly defined in the analysis, can be introduced as constant damping for all frequencies and interpolated damping by frequency. Constant damping for all frequencies is defined through the factors of mass and stiffness participation (α and β), so that the damping matrix $[C]$ can be calculated using expression (1).

3. Aspects of damping modelling in structural analysis

3.1 *Spectral-modal analysis*

Damping can be introduced into *spectral-modal analysis* by using: material damping, link element damping and analysis damping. Figure 2 shows the flowchart of introducing damping into *spectral-modal analysis*.

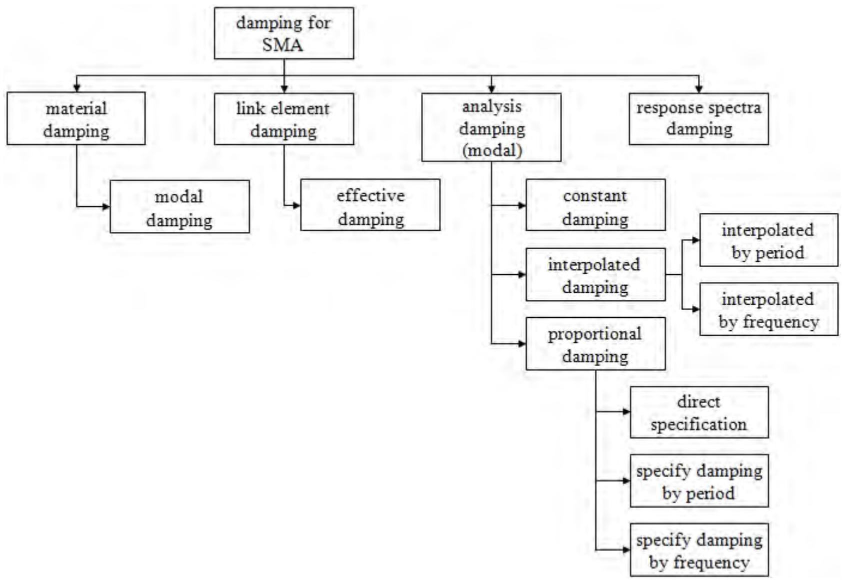

Fig. 2. Flowchart of introducing damping into *spectral-modal analysis*

Material damping is introduced as a modal damping, while the link element damping is introduced as effective damping. The damping which is directly defined in the analysis is introduced as: constant damping, interpolated damping, using the factors of mass and stiffness participation, whereby the latter can be introduced by using the factors of mass and stiffness participation (α and β) as a function of the vibration period of the first and second eigenform T_1 and T_2, and as a function of frequencies f_1 and f_2. On the other hand, when generating the response spectrum, damping is introduced through the relative damping coefficient ξ_{rs}. However, overall damping in the *spectral-modal analysis* is defined through the cumulative relative damping coefficient ξ, so that the response spectrum curve is corrected according to [8]:

$$S_a = S_{a,rs} \frac{2.31 - 0.41 \cdot \log \xi}{2.31 - 0.41 \cdot \log \xi_{rs}}, \tag{10}$$

where S_a is the corrected spectral acceleration corresponding to damping ξ, and $S_{a,rs}$ is the initial spectral acceleration corresponding to damping ξ_{rs}.

3.2 *Non-linear static pushover analysis*

In *non-linear static pushover analysis* damping is not introduced before the calculation; instead, it is subsequently defined after the structure is calculated in *target displacement analysis*. Figure 3 shows the flowchart of introducing damping into *non-linear static pushover analysis*.

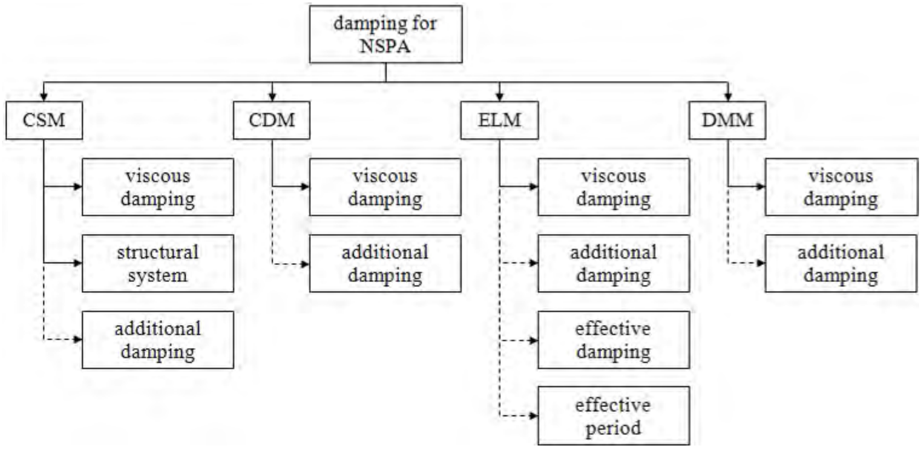

Fig. 3. Flowchart of introducing damping into *non-linear static pushover analysis*

Damping is introduced through a global coefficient which can take into account both viscous and hysteretic damping. Depending on the type of *target displacement analysis*, the following options are possible:

- *capacity spectrum method* (CSM):
  Damping is introduced through a global damping coefficient as inherent and additional damping, but it can also be affected through the type of the structural system. For the level of target displacement d_t, which is determined by iterations, the overall effective damping in the system ξ_{eff} is obtained from [9]:

$$\xi_{eff} = \kappa\xi_h + \xi_v = \frac{63.7\kappa\left(S_{a,y}S_{d,t} + S_{d,y}S_{a,t}\right)}{S_{a,t}S_{d,t}} + \xi_v, \tag{11}$$

  where ξ_v is the relative (viscous) damping coefficient (5%), ξ_h is the hysteretic damping coefficient (shown as equivalent viscous damping), $S_{a,y}$ is spectral acceleration at the yield point shown in the format of acceleration-displacement response spectra (ADRS), $S_{a,t}$ is spectral acceleration for the level of target displacement, $S_{d,y}$ is spectral displacement at the yield point, $S_{d,t}$ is spectral displacement for the level of target displacement, κ is the coefficient that takes into account how well the structure's hysteretic model is approximated by the bilinear hysteretic model.

- *coefficient displacement method* (CDM):
  Damping is introduced through the effective damping coefficient which is used in generating the response spectrum. In fact, this is a viscous damping, while a hysteretic damping is determined from the calculation, but additional damping may also be introduced over this coefficient [10].
- *equivalent linearization method* (ELM):
  Damping is introduced through the global damping coefficient (basic and additional damping), but effective damping can be defined as an alternative solution, shown through the relative damping coefficient for the system's hysteretic response [11].
- *displacement modification method* (DMM):
  Damping is introduced through the effective damping coefficient, which is used in generating the response spectrum. Essentially, this is a viscous damping, while a hysteretic is determined from the calculation, but additional damping may also be introduced over this coefficient [11].

3.3 *Linear dynamic (modal) analysis*

Damping in *linear dynamic (modal) analysis* can be introduced by using: material damping, link element damping and damping in the analysis. Figure 4 shows the flowchart of introducing damping into *linear dynamic (modal) analysis*. Material damping is introduced as a modal damping, while the link element damping is introduced as effective damping. Damping that is directly defined in the analysis is introduced as: constant damping, interpolated damping, and using the factors of mass and stiffness participation, whereby the latter can be introduced using: factors of mass and stiffness (α and β) participation as a function of the vibration period and a function of frequency.

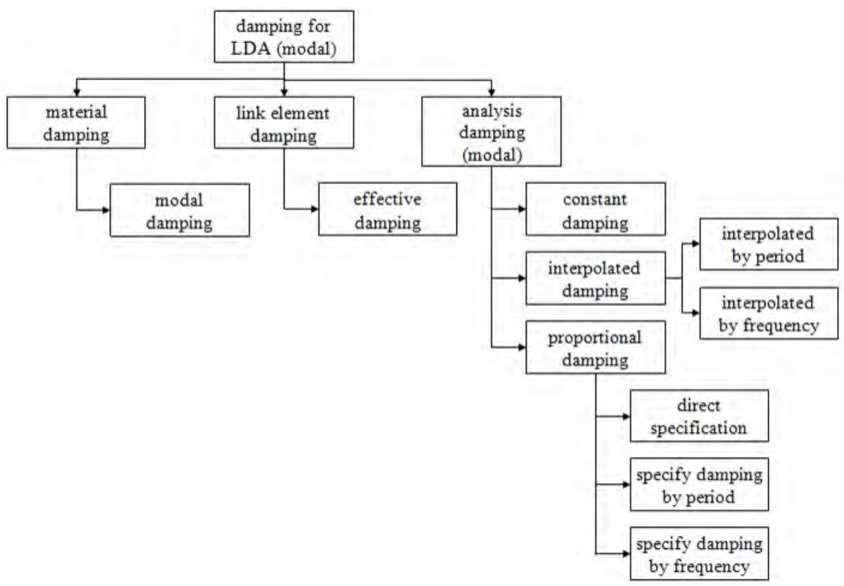

Fig. 4. Flowchart of introducing damping into *linear dynamic (modal) analysis*

3.4 *Non-linear dynamic (modal) analysis*

Damping in *non-linear dynamic (modal) analysis* can be introduced by using: material damping, damping in the non-linear behaviour of link elements and damping in the analysis. Figure 5 shows the flowchart of introducing damping into *non-linear dynamic (modal) analysis*. Material damping is introduced as modal damping, while damping of link elements is introduced taking into account the predefined parameters for non-linear damping and the development hysteretic behaviour. Damping that is directly defined in the analysis is introduced in the same way as in *linear dynamic (modal) analysis*: constant damping, interpolated damping and using the factors of mass and stiffness participation.

The efficiency of this analysis is reflected in separating the force vector originating from the link element with non-linear behaviour from the matrix of elastic stiffness and damping.

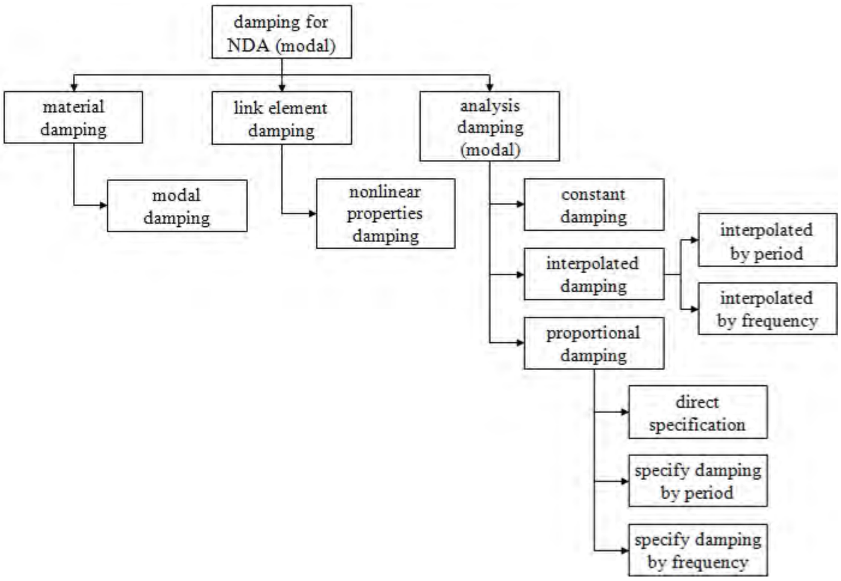

Fig. 5. Flowchart of introducing damping into *non-linear dynamic (modal) analysis*

3.5 *Linear dynamic (numerical integration) analysis*

In *linear dynamic (numerical integration) analysis* damping can be introduced by using: material damping, link element damping and damping in the analysis. Figure 6 shows the flowchart of introducing damping into *linear dynamic (numerical integration) analysis*. Material damping is introduced as viscous damping, while the link element damping is introduced as effective damping. Damping that is directly defined in the analysis is introduced by using: the factors mass and stiffness participation (α and β) as a function of the vibration period and a function of frequency.

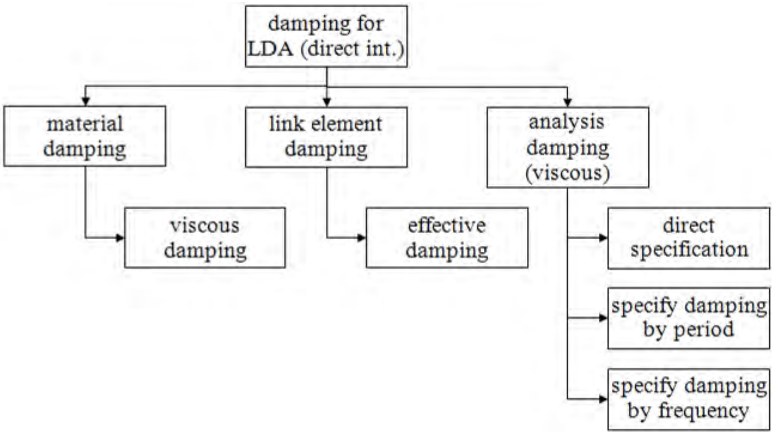

Fig. 6. Flowchart of introducing damping into *linear dynamic (numerical integration) analysis*

3.6 *Non-linear dynamic (numerical integration) analysis*

In *non-linear dynamic (numerical integration) analysis* damping can be introduced by using: material damping, link element damping and damping in the analysis. Figure 7 shows the flowchart of introducing damping into *non-linear dynamic (numerical integration) analysis*. Material damping is introduced as viscous damping, while the link element damping is introduced taking into account the predefined parameters for non-linear damping and the development hysteretic behaviour. Damping that is directly defined in the analysis is introduced in the same way as in *linear dynamic (numerical integration) analysis*. In the case of pronounced non-linear behaviour due to the constant reduction of the system stiffness, a constant reduction of damping occurs, although it lacks any physical justification [7]. Then the damping matrix is the best to be formed at the beginning of the calculation, as proportional to initial linear stiffness matrix while neglecting the member which is proportional to the mass matrix. This can be explained by the fact that effects of hysteretic dissipation in non-linear systems are more dominant than the effects of viscous damping, which is more expressed in linear systems. Eliminating the member which is proportional to the mass matrix allows higher eigenforms to be damped more than the lower eigenforms.

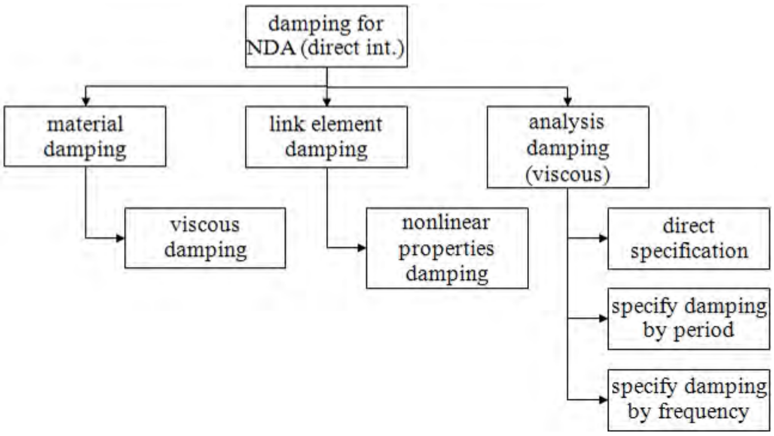

Fig. 7. Flowchart of introducing damping into *non-linear dynamic (numerical integration) analysis*

3.7 ***Steady-state analysis and power spectral density analysis***

Damping can be introduced into *steady-state* and *power spectral density analysis* by using: material damping, link element damping and damping in the analysis. Figure 8 shows the flowchart of introducing damping into *steady-state* and *power spectral density analysis*. Material damping is introduced as hysteretic damping, while the link element damping is introduced as effective damping and damping in frequency dependent link elements. Damping which is directly defined in the analysis is introduced as constant (using the factors of mass and stiffness participation α and β) and interpolated damping (interpolation by frequency and factors of mass and stiffness participation, α and β).

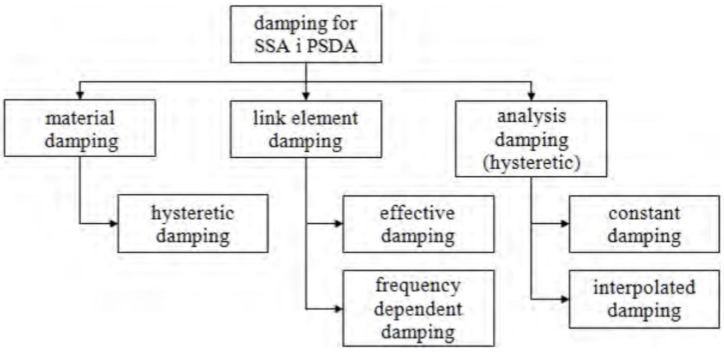

Fig. 8. Flowchart of introducing damping into *steady-state* and *power spectral density analysis*

In order to address the problem in the frequency domain, the complex impedance matrix can be represented as a function of frequency as [7]:

(12) $$[\overline{K}(\omega)] = [K(\omega)] - \omega^2[M] + i\omega[C(\omega)].$$

For problems in the frequency domain, the damping matrix is usually represented as a hysteretic damping matrix according to (3). When damping is interpolated by frequency and factors of mass and stiffness participation (α and β), then the damping matrix becomes:

(13) $$[D(\omega)] = \alpha(\omega)[M] + \beta(\omega)[K].$$

Conclusion

At the conceptual level, the introduction of damping into structural analysis should be considered as a function of the type of analysis (linear or non-linear analysis), that is, as a function of the domain type in which the system response is analyzed (capacitive, time or frequency domain). Since the modelling of damping is one of the biggest issues in structural analysis, this research suggests the engineering community a flowchart based solution to eliminate the existing dilemmas about the modelling of damping. The traditional approach of the introduction of damping into structural analysis is based on the relative damping coefficient ξ. However, one should know that taking into account the relative damping coefficient is not equivalent to introducing (hysteretic) damping into the analysis using factors of mass and stiffness participation α and β, which were defined for the value of the relative damping coefficient. Also, when introducing damping into the analysis, care should be taken to be aware of the existence of damping in the system, not to duplicate the damping values or introduce the same type of damping but in two different ways.

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