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COMPARISON OF DISTRIBUTED PLASTICITY APPROACHES FOR INELASTIC ANALYISIS OF REINFORCED CONCRETE FRAMES

Abstract

Following the emergence and establishment of performance-based design as a widely recognized and applied practice, numerous significant advances in computational techniques have been made. Arguably, nonlinear methods of analysis – static or dynamic – provide the most detailed and accurate insight into behavior of the structure which is designed or assessed. While the current codes of practice permit their use, provisions are vague in some aspects, leaving many decisions and assumptions up to the engineer. This paper aims to expand the relatively limited experience with implementation of different distributed plasticity modeling approaches through a case study based on experimental data. Some particulars of the behavior of structural elements made of strain-softening materials, and of their corresponding numerical models, are briefly introduced. Case study is conducted to provide an overview on the procedures for inelastic analysis of reinforced concrete frames through a side-by-side comparison of four alternative approaches for modeling of RC beams and columns with one-dimensional distributed plasticity finite elements with fiber-discretized cross sections. The analyzed structure consists of a reinforced concrete column with a fixed base, which is, under varying axial loads, subjected to monotonic and cyclic lateral displacements applied at its top. Models comprised of force-based elements with different arrangements of integration points, as well as of displacement-based elements, are compared. Advantages, limitations and special considerations which need to be taken into account for each of the approaches are demonstrated through the example. Influence of concrete confinement is studied by analyzing another column with a different arrangement of transverse reinforcement. Global structural response is evaluated by comparison of pushover and cyclic curves for different models. Influence of modeling approach on objectivity of the local response and the corrections which need to be made for certain approaches are showcased by moment-rotation curves of the yielded section at the column base. Considerations and findings presented in the paper, along with other published case studies, can be used for reference when preparing a computational model for inelastic analysis of a reinforced concrete structure.

Keywords

Nonlinear analysis, performance-based design, seismic assessment, finite elements

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1. INTRODUCTION

Inelastic analysis of reinforced concrete structures poses a particular challenge due to the strain-softening behavior of concrete, among other reasons. With strain-hardening materials (e.g. steel), the results converge as the model is refined. Fiber-section models are especially suitable for such analyses, since they can capture spread of plasticity along the member. However, if the material exhibits reduction in strength as it is loaded, plasticity tends to concentrate at a single cross section. Thus, the response of such models is inherently dependent on the weight assigned to the integration point at that location, which depends on modeling assumptions such as the adopted integration scheme or member subdivision into finite elements.

Two formulations of finite elements are usually applied for such analyses: displacement-based and force-based. Displacement- or stiffness-based formulation of the finite element method is widely used in computational mechanics. The displacement field is approximated by shape functions which assume constant axial deformation and linear curvature along the element. To capture variable axial deformation and curvature distribution of a higher order, member needs to be subdivided into multiple finite elements. In the Force- or flexibility-based approach, force field is represented by interpolation functions which are constant for axial force and linear for bending moment. Response accuracy of these elements can be improved if the numerical integration error is reduced, which can be achieved by adjusting the number and positions of integration points along the element. Neuenhofer and Filippou [1] gave a detailed comparison illustrated through examples.

Since the geometrically nonlinear force-based line element was introduced by Neuenhofer and Filippou [2], multiple attempts at improvement have been made. Some of the novel formulations were developed specifically to overcome the difficulties with numerical analysis of reinforced concrete members. Force-based element with modified Gauss-Radau integration developed by Scott and Fenves [3], or the element based on gradient inelastic beam theory recently proposed by Sideris and Salehi [4], are such examples.

Scope of this paper is to expand the experience with nonlinear analysis of reinforced concrete members characteristic to buildings and similar structures through a direct comparison and evaluation of four alternative distributed plasticity modeling approaches. Beams and columns in such structures are normally slender and flexure-critical, with rotational plastic hinges forming at the element ends, where the bending moment is greatest. As a baseline against which the numerical results are validated, an experimental dataset of cyclic load tests of rectangular, relatively slender columns, conducted by Saatcioglu and Grira [5], was sourced from the PEER Structural Performance Database [6]. Monotonic and cyclic pushover analyses were conducted on four different models for each of the selected specimens, the results of which are presented and compared in the paper. OpenSees computational framework [7] was utilized in the case study since it provides a comprehensive set of force- and displacement-based line elements, with complete control over the numerical integration method which is employed [8].

2. CASE STUDY – EXPERIMENTAL DATASET

Saatcioglu and Grira [5] conducted a series of cyclic load tests on axially loaded reinforced concrete columns. Distance from the column base to the point of lateral loading is 1.645 m for all specimens. Dimensions of the cross sections are 350×350 mm, with 29 mm of clear concrete cover.

Confinement is provided by three-legged hoops. Concrete compressive strength is noted as 34 MPa. Results are taken from three specimens – BG-1, BG-2 and BG-3 – the configuration of which is shown in Table 1. Longitudinal reinforcement of all three specimens is identical, consisting of eight 19.5 mm diameter bars. Transverse reinforcement is of the same diameter for all three columns (9.5 mm), while the spacing varies (Table 1).

Table 1. Summary of the test specimens [5]

Specimen	Axial load (kN)	Axial load ratio	Hoop spacing (mm)
BG-1	1,782.0	0.428	152
BG-2	1,782.0	0.428	76
BG-3	831.0	0.2	76

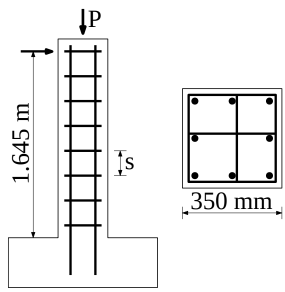

Figure 1. Test setup by Saatcioglu and Grira [5]

3. DESCRIPTION OF THE NUMERICAL MODELS

Some general simplifications are adopted: neither the strain-penetration effects, nor the shear behavior are represented in the model. Strain-penetration (or bar-slip) effects are, regardless of the computational model used to represent the RC member, generally dealt with in the same way – by adding rotational springs with assigned stress-slip relations to the member ends. Since the evaluated columns are relatively slender and experience flexural failure, shear behavior is not incorporated into the model. Elastic shear deformation was found to have negligible effect on the model response.

3.1. MATERIALS

Concrete is modeled by Kent-Park stress-strain relation [9] with parameters taken from the constitutive model by Saatcioglu and Razvi [10], which was found to be quite generous in terms of ultimate strain for both unconfined and confined concrete, compared to modified Kent-Park [11] and Eurocode 8-3 [12] models. Saatcioglu-Razvi model provides the closest prediction of cyclic behavior in comparison to the experimental data. Although the ascending branch of the Kent-Park model is not well-suited to the values of Saatcioglu-Razvi model (Figure 2a), this formulation was kept for its simplicity and suitability for regularization (needed in the first modeling approach) due to the linear post-peak relation. Unconfined concrete is assumed to spall after reaching the ultimate strain, hence the post-peak branch drops to zero.

Bilinear model with gradual transition between the branches is used for reinforcing steel (Figure 2b). Strain hardening ratio of 0.01 is adopted. Isotropic and kinematic hardening are not being taken into account.

Material properties are directly incorporated into the models through fiber-discretized cross sections (Figure 3a), in which a uniaxial constitutive model of appropriate material is assigned to every fiber.

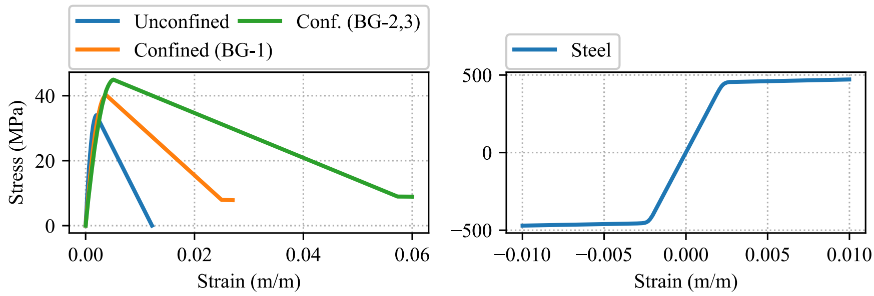

Figure 2. Constitutive models for (a) concrete and (b) steel

3.2. PLASTIC HINGE LENGTH

Length of the plastic hinge can be determined from empirical equations, which are based on observations from experiments. Equation by Paulay and Priestley [13] gives the hinge length of 0.327 m, while length obtained with the Eurocode 8-3 [12] equation is 0.292 m. The former value is adopted, although both are within the expected range. This value is used in distributed plasticity approaches to define length of the localization zone (Approaches 2, 3 and 4) or to obtain regularized constitutive models (Approach 1).

3.3. DIFFERENT MODELING APPROACHES

Approach 1: Force-based element with Gauss-Lobatto integration method. Gauss-Lobatto integration scheme places integration points at the element ends (Figure 3b), which makes it favorable for inelastic analysis. Since the locations and weights of the integration points are fixed (although they can be somewhat adjusted by changing the number of points), objectivity of the response must be provided by regularization of the constitutive models [14]. Regularization of concrete is performed by the method proposed by Pozo [15]. Dependence of the section curvature demand on the plastic zone length is accounted for by modification of the ultimate strain ε_{cu}, so that the concrete crushing energy (area under the stress-strain curve) remains constant for different curvature demands. Modified ε_{cu} for specimen BG-1 is increased to 0.046 and for BG-2 and BG-3 to 0.073, while the strength of unconfined concrete drops to zero at a strain of 0.038. As per Pozo's recommendation, steel stress-strain relation is not regularized.

Approach 2: Force-based element with plastic hinge integration method. The second approach to the localization issue is to adjust the length assigned to the integration point where plasticity occurs, so it matches the length of a real-life plastic hinge, where concrete crushing and steel yielding would be taking place. Element formulation by Scott and Fenves [2] with modified Gauss-Radau integration is used in the model (Figure 3c). This element formulation is commonly referred to as „beam with hinges" (BWH).

Approach 3: Mesh of displacement-based elements. Refinement of the integration scheme generally does not improve the response of displacement-based elements, so a two-point Gauss-Legendre integration is commonly used. The member is rather subdivided into a mesh of finite elements. In order to avoid material regularization, the finite element mesh consists of three DB elements, with the bottom one being twice as long as the assumed plastic hinge (Figure 3d). Meshes produced this way are normally coarse, which can lead to overestimation of stiffness and strength. Alternatively, finer meshes may also be applied, in which case the constitutive models would have to be regularized, as in the Approach 1.

Approach 4: Gradient inelastic beam theory. This force-based element formulation, developed by Sideris and Salehi [4], is used in a similar manner as the BWH element, but offers certain computational advantages. It requires a relatively dense layout of integration points (Figure 3e). For the analyzed models, characteristic length l_c, which defines the localization zone, has to be increased 1.8 times to match the response of the BWH element, so the length of 0.59 m is adopted.

Table 2. Summary of the computational models

Approach	Formulation	Elements per member	IPs per element	Integration rule
A1	force-based	1	5	Gauss-Lobatto
A2	force-based	1	6	Modified Gauss-Radau
A3	disp.-based	3	2	Gauss-Legendre
A4	force-based	1	9	Simpson

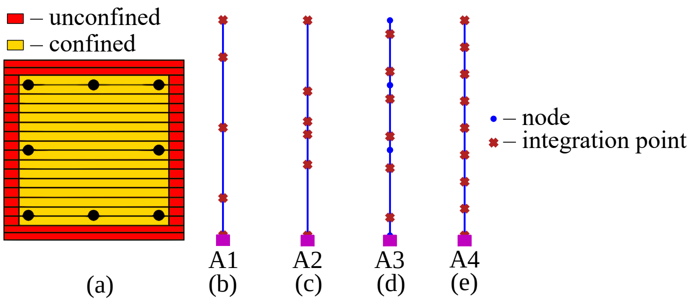

Figure 3. (a) Fiber-discretized cross section; (b-e) Models for different approaches

4. RESULTS AND DISCUSSION

Pushover analysis (Figure 4) outputs consistent results for all three specimens, where A2 and the adjusted A4 models behave in a virtually identical manner, A3 overestimates yield point due to coarse mesh, and A1 deteriorates at an increased rate.

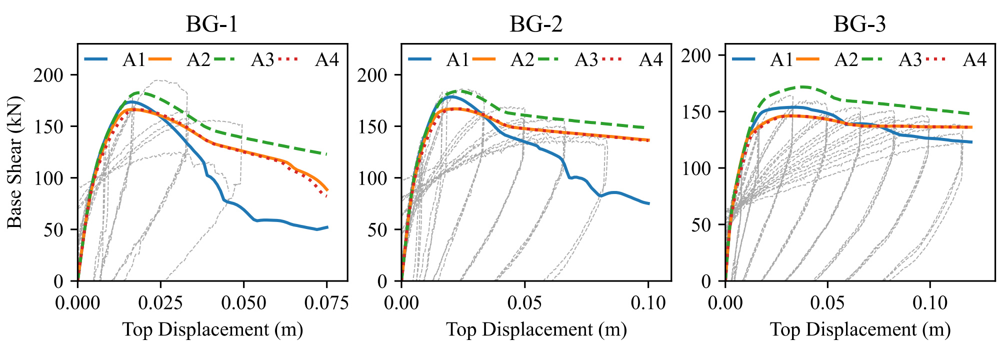

Figure 4. Pushover curves

Moment-curvature response of cross section at the base is consistent with monotonic pushover, except for the A1 model (Figure 5). To overcome the loss of objectivity of the Approach 1, correction proposed by Coleman and Spacone [14] is applied. Post-elastic portion of the moment-curvature relation is scaled according to the adopted plastic hinge length and weight of the integration point (Figure 6).

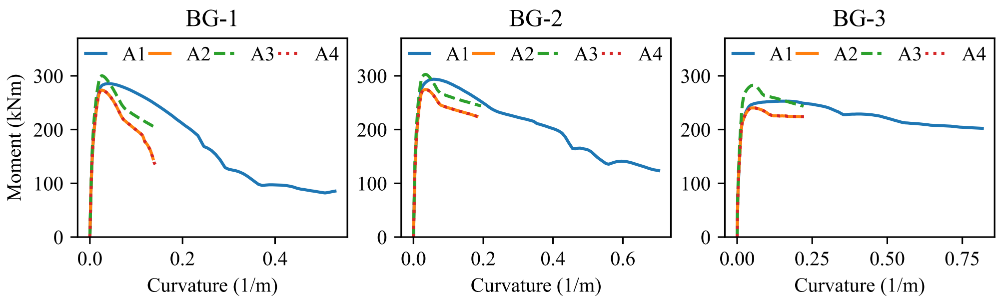

Figure 5. Moment-curvature for static pushover

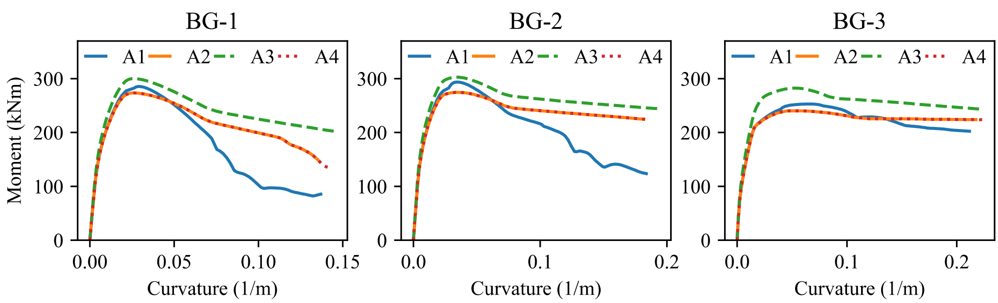

Figure 6. Moment-curvature with corrections for Approach 1

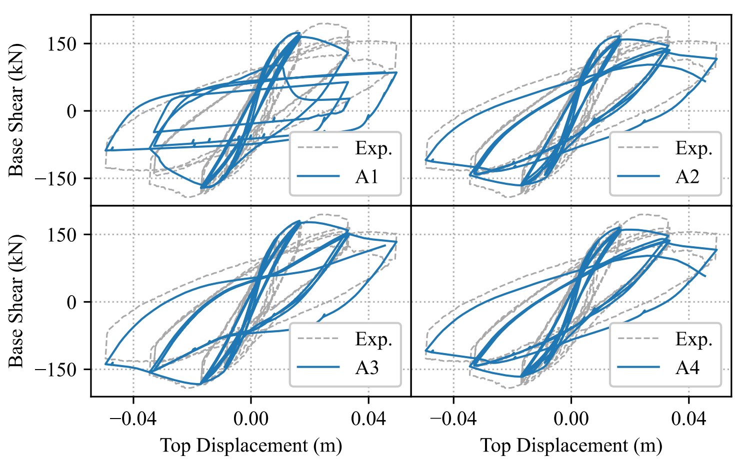

Figure 7. Hysteresis loops of the BG-1 specimen

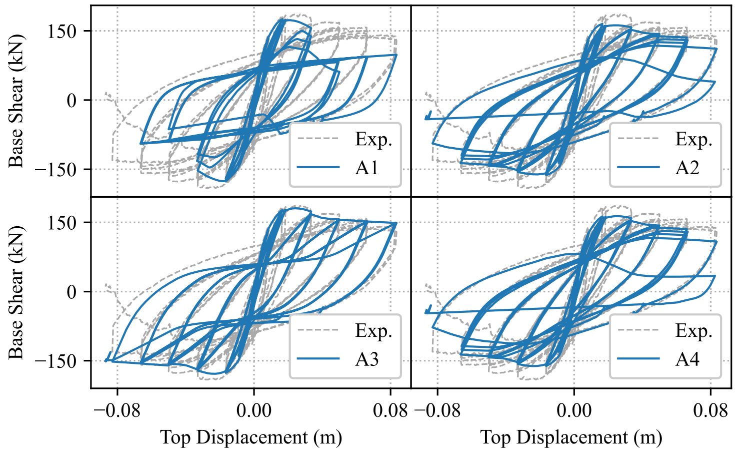

Figure 8. Hysteresis loops of the BG-2 specimen

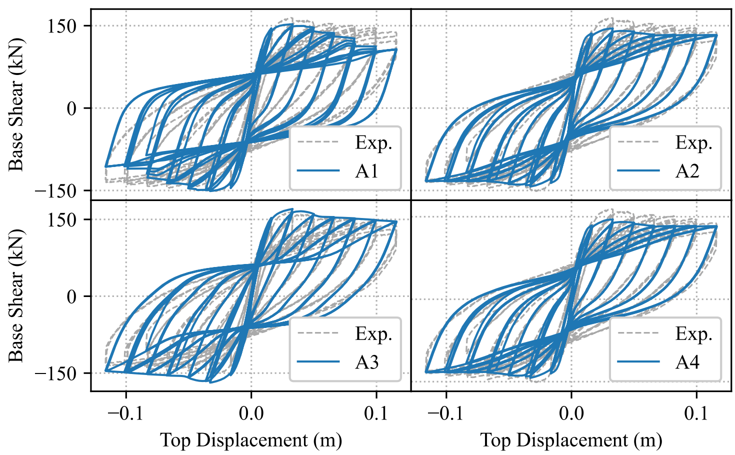

Figure 9. Hysteresis loops of the BG-3 specimen

Cyclic pushover analysis (Figures 7–9) points to similar conclusions. Modeling approaches 2 and 4 give the best agreement with the experiment, going as far as predicting the cycle in which the failure occurs. Model A3 does not capture cumulative damage equally well and A1 deteriorates prematurely. Slight underestimation of the yield capacity by the A2 and A4 models may be attributed, among several potential factors, to the simplified steel constitutive relation which captures neither early onset of strain hardening, nor isotropic hardening under cyclic load.

5. CONCLUSIONS

In the presented case study, Approach 1 with material regularization produced reasonably accurate results only for a lower axial load. Regularization techniques for columns are still based on limited dataset and experience and have a potential to introduce errors, as can be seen with specimens BG-1 and BG-2.

At the current state-of-the-art, force-based elements with adjustable localization zones (such as those presented in Approaches 2 and 4) provide the most straightforward workflow with the least possibility of error. Force-based elements based on gradient theory (Approach 4) offer convenience similar as the BWH elements (Approach 2) while being more numerically robust for different member geometries, but should be further verified against experimental data and other computational models in order to establish reliable recommendations for adoption of the characteristic length l_c.

Despite the good agreement with experimental results that was achieved with previously described constitutive model for concrete, more conservative, as well as consistently implemented models should be used in engineering practice.

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